KEY

First Name:______________________________ Last Name:______________________________

L16&17 – Worksheet –Unit Review /30 marks

Part 1: Gravitational Forces & Fields in 1-Dimension

Part 2: Gravitational Forces & Fields in 2-Dimensions

Part 3: Algebraic and Graphical Gravity Questions

Part 4: Centripetal Speed and Acceleration

Part 5: Circular Motion (Horizontal Systems)

Part 6: Circular Motion (Vertical Systems)

Part 7: Kepler's Laws

Part 8: Algebraic and Graphical Circular Motion Questions

■ KEY ■

Part 1: Gravitational Forces & Fields in 1-Dimension

Q1: If a 5000kg object was located 3m from a 10,000kg object, what is the gravitational force between these two objects?

- [✓ a.] 3.71 x 10^{-4} N
- b. 5.56 x 10^{-4} N
- c. 1.11 x 10^{-3} N
- d. 5.45 x 10^{7} N

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{(6.67\times10^{-11})(5000)(10{,}000)}{(3)^2}$$

$$F_g = 3.71\times10^{-4}\,N$$

Q2: Two identical objects of mass 5000kg are placed a distance apart. They experience a gravitational force of attraction of 1.00 x 10^{-6} N. The distance between their center of masses is ***a.bc*** x 10^{d} m, where ***a, b, c,*** and ***d*** are ____, ____, ____, and ____.

(Record your ***four-digit*** *answer in the Numerical Response boxes below)*

4	0	8	1

$$F_g = \frac{Gm_1m_2}{r^2} \Rightarrow r^2 = \frac{Gm_1m_2}{F_g}$$

$$r^2 = \frac{(6.67\times10^{-11})(5000)(5000)}{1.00\times10^{-6}}$$

$$r^2 = 1667.5$$

$$r = 40.835\ m$$

$$r \approx 4.08\times10^{1}\,m$$

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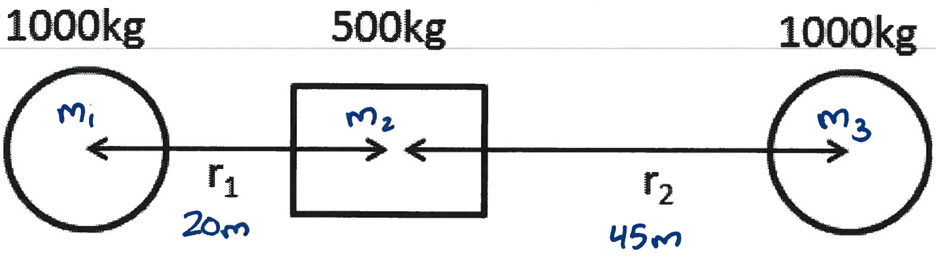
Q3: If $r_1 = 20$m, and $r_2 = 45$m, calculate the acceleration (magnitude and direction) of the 500kg block.
(2 marks)



$$Fg_{12} = \frac{Gm_1m_2}{r^2} = \frac{(6.67\times10^{-11})(1000)(500)}{(20)^2} = 8.3375\times10^{-8} \text{ N [left]}$$

$$Fg_{23} = \frac{Gm_2m_3}{r^2} = \frac{(6.67\times10^{-11})(500)(1000)}{(45)^2} = 1.6469\times10^{-8} \text{ N [right]}$$

$$\vec{F}_{net} = \vec{F}_{12} + \vec{F}_{23} \qquad - \longleftrightarrow +$$

$$= (-8.3375\times10^{-8}) + (1.6469\times10^{-8})$$

$$= -6.6906\times10^{-8} \text{ N [right]}$$

$$= 6.6906\times10^{-8} \text{ N [left]}$$

$$\vec{a} = \frac{F_{net}}{m} = \frac{6.6906\times10^{-8} \text{ N [L]}}{500\text{kg}} = 1.3381\times10^{-10} \text{ m/s}^2 \text{ [L]}$$

$$\boxed{\vec{a} = 1.3381\times10^{-10} \text{ m/s}^2 \text{ [Left]}}$$

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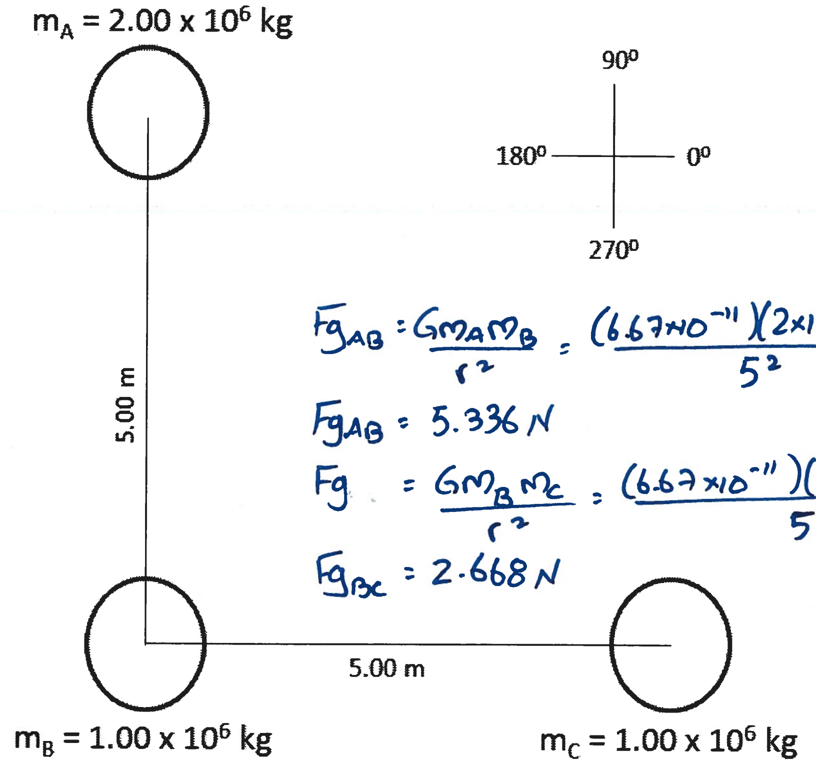
Part 2: Gravitational Forces & Fields in 2-Dimensions

Use the following information to answer Q4:

Three objects are arranged at right angles to each other, as depicted below.



$$Fg_{AB} : \frac{Gm_A m_B}{r^2} = \frac{(6.67\times10^{-11})(2\times10^6)(1\times10^6)}{5^2}$$

$Fg_{AB} = 5.336\ N$

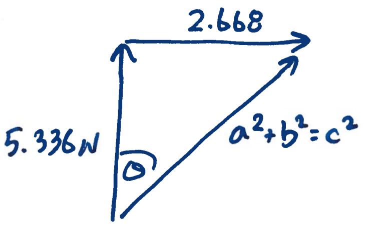
$$Fg = \frac{Gm_B m_C}{r^2} = \frac{(6.67\times10^{-11})(1\times10^6)(1\times10^6)}{5^2}$$

$Fg_{BC} = 2.668\ N$

Q4: The net force acting on m_B is ***a.b*** Newtons [***ef*** degrees], where ***a, b, e,*** and ***f*** are ____, ____, ____, and ____.

*(Record your **four-digit** answer in the Numerical Response boxes below)*

6	0	6	3



⇒

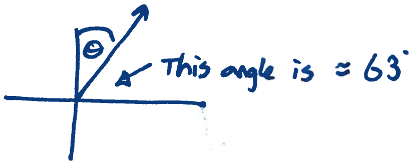
$a^2+b^2=c^2$

$c = 5.9658 \approx 6.0$

$\tan\theta = \frac{o}{a}$

$\theta = \tan^{-1}\left(\frac{2.668}{5.336}\right)$

$\theta = 26.565°$



$\vec{F}_{net} = 6.0N\ [63°]$

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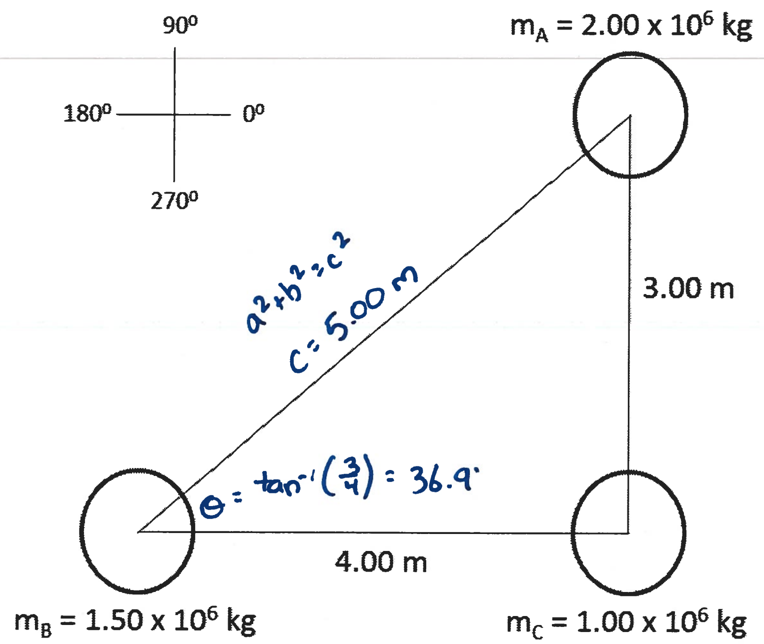
Q5: What is the net force acting on m_B? *(3 marks)*



$a^2+b^2=c^2$

$c = 5.00$ m

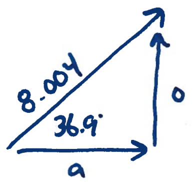
$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.9°$

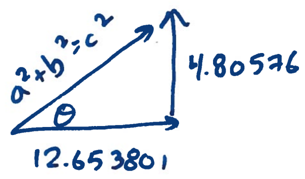
$F_{AB} = \frac{Gm_Am_B}{r^2} = \frac{(6.67\times10^{-11})(1.5\times10^{6})(2\times10^{6}}{5^2}$

$F_{AB} = 8.004N$

$F_{BC} = \frac{Gm_Bm_C}{r^2} = \frac{(6.67\times10^{-11})(1.5\times10^{6})(1\times10^{6})}{4^2}$

$F_{BC} = 6.253125N$

	x-comp	y-comp	Unit Vector (Optional)
	$\cos\theta = \frac{a}{h}$ $a = h\cos\theta$ $+6.400676$	$\sin\theta = \frac{o}{h}$ $o = h\sin\theta$ $+4.80576$	$\vec{F}_{AB} = 6.400676\hat{i} + 4.80576\hat{j}$
6.253125N →	$+6.253125N$	$+0N$	$\vec{F}_{BC} = 6.253125\hat{i} + 0\hat{j}$
$\vec{F}_{net}$	$+12.653801$	$+4.80576$	$\vec{F}_{net} = 12.653801\hat{i} + 4.80576\hat{j}$



$a^2+b^2=c^2$

$c = 13.5356...$

$c \approx 13.5N$

$\tan\theta = \frac{o}{a}$

$\theta = \tan^{-1}\left(\frac{4.80576}{12.653801}\right)$

$\theta = 20.796°$

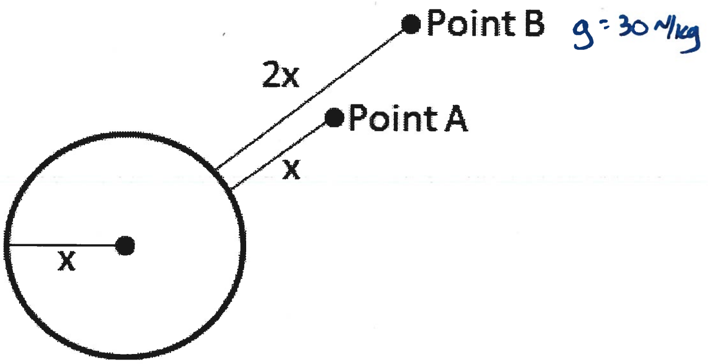
$\boxed{\vec{F}_{net} = 13.5N\ [20.8°]}$

■ KEY ■

Part 3: Algebraic and Graphical Gravity Questions

Use the following information to answer Q6-Q7:

An Astronaut of mass 80kg is located at Point B. If the gravitational field strength experienced by the Astronaut at this location is 30 N/kg.



Q6: The Force of Gravity acting on the Astronaut at Point B is ***a.bc*** x 10^d N, where ***a, b, c,*** and ***d*** are ___, ___, ___, and ___.

*(Record your **four-digit** answer in the Numerical Response boxes below)*

2	4	0	3

$$F_g = mg$$
$$= (80\,kg)(30\,N/kg)$$
$$= 2400\,N$$
$$= 2.40 \times 10^3\,N$$

Q7: What is the acceleration due to gravity on the surface of the planet, in m/s^2?

*(Record your **three-digit** answer in the Numerical Response boxes below)*

2	7	0	

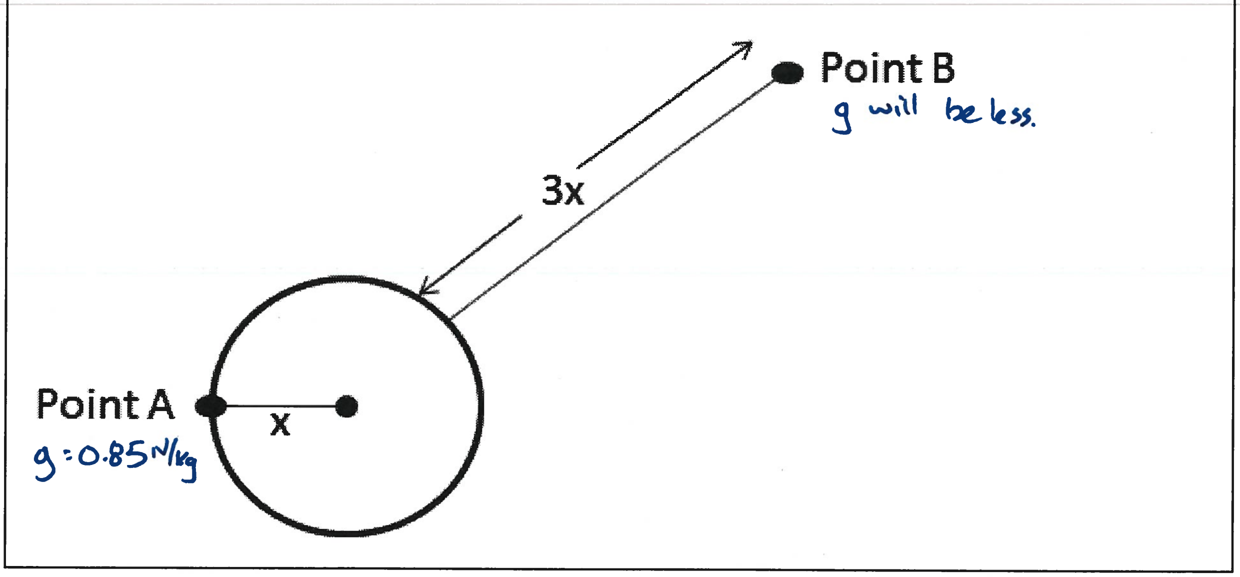
Going from 3x distance to 1x distance.
So now 1/3 distance. Expect g to increase.

$$g = \frac{Gm}{r^2} = 30\,N/k \qquad g_{new} = \frac{Gm}{(\frac{1}{3}r)^2} = \frac{Gm}{\frac{1}{9}r^2} = 9\left(\frac{Gm}{r^2}\right)$$
$$= 9(30)$$
$$= 270\,m/s^2$$

■ KEY ■

Use the following information to answer Q8-Q9:

An Astronaut of mass 60kg is located at Point A. The gravitational field strength experienced by the Astronaut at this location is 0.85 N/kg.



Q8: What is the Force of Gravity acting on the Astronaut at Point A?

(Record your ***three-digit*** *answer in the Numerical Response boxes below)*

5	1	.	0

$$F_g = mg$$
$$= (60\,kg)(0.85\,N/kg)$$
$$= 51\,N$$
$$= 51.0\,N$$

Q9: The acceleration due to gravity at Point B is ***a.bc*** x 10^{-d} m/s^2, where ***a, b, c,*** and ***d*** are ____, ____, ____, and ____.

(Record your ***four-digit*** *answer in the Numerical Response boxes below)*

5	3	1	2

$g = 0.85\,N/kg$

$$g_{new} = \frac{Gm}{(4r)^2} = \frac{Gm}{16r^2} = \frac{1}{16}\left(\frac{Gm}{r^2}\right)$$
$$= \frac{1}{16}(0.85)$$
$$= 0.053125$$
$$\approx 5.31\times10^{-2}\,N/kg$$

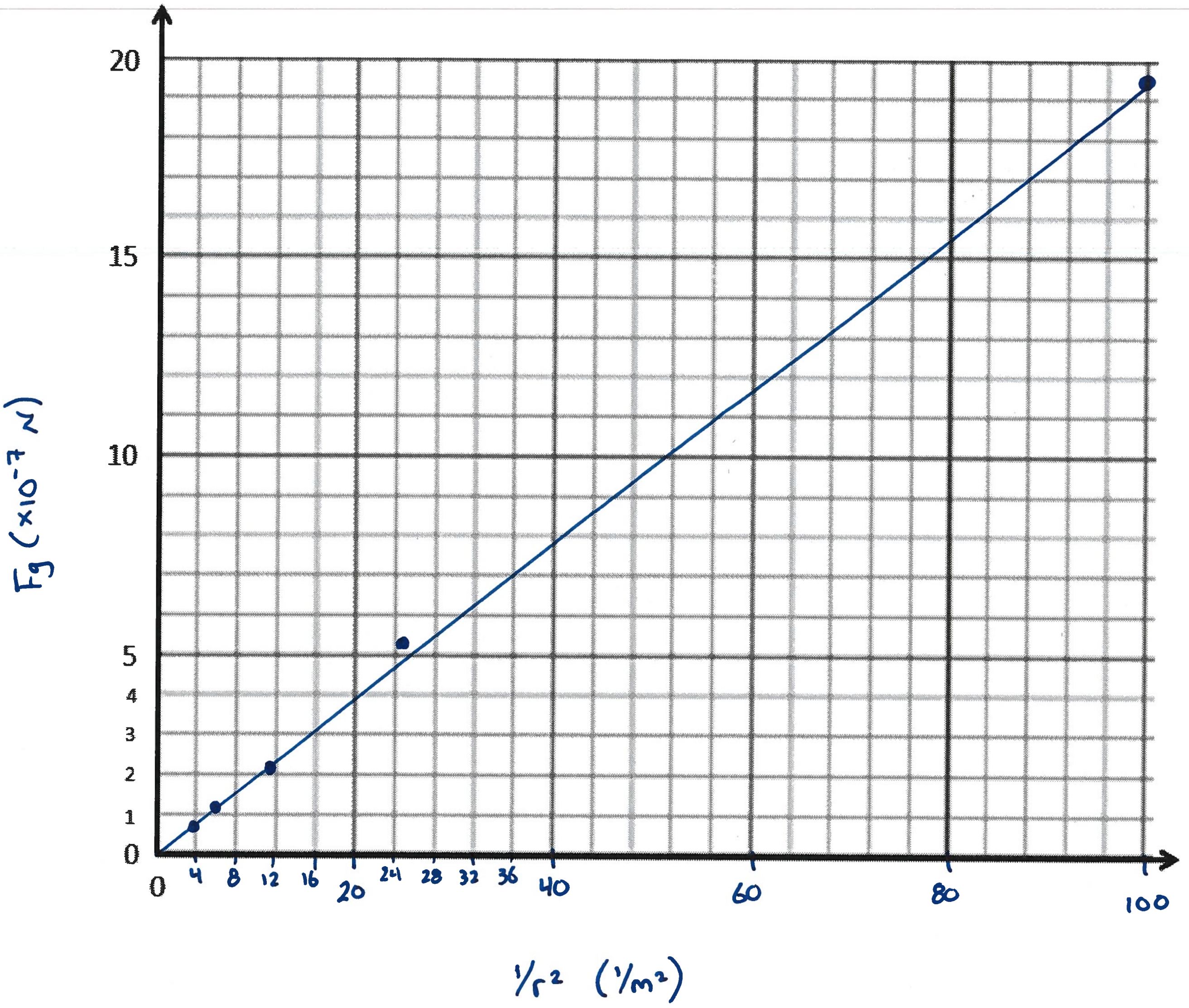
■ KEY ■

Q10: The gravitational force of attraction between a 10kg object and another object of unknown mass is plotted below for various distances. Linearize the data, graph it, and use the slope to determine the magnitude of the unknown mass. *(2 marks)*

Radius, r (meters)	Force of Gravity, F_g (Newtons)	Inverse radius squared, $1/r^2$ ($1/m^2$)
0.1	19.5×10^{-7}	100
0.2	5.3×10^{-7}	25
0.3	2.1×10^{-7}	$11.\overline{1}$
0.4	1.2×10^{-7}	6.25
0.5	0.7×10^{-7}	4

KEY

Linearized Gravitational Force



$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{19.5\times10^{-7}}{100} = 1.95\times10^{-8}$$

$$F_g = \frac{Gm_1m_2}{r^2} \qquad F_g = (Gm_1m_2)\frac{1}{r^2} + 0$$

$$y = (\text{Slope})\ x \ + b$$

$$\Rightarrow \text{Slope} = Gm_1m_2$$

$$1.95\times10^{-8} = (6.67\times10^{-11})(10)m_2$$

$$\boxed{m_2 = 29.2 \text{ kg}}$$

KEY

Part 4: Centripetal Speed and Acceleration

Use the following information to answer Q11-Q12:

The Vitamix © 5200 Blender, depicted below, is rated at 1380W. It has a blade of radial length 3.8cm (0.038m), where the tip of the blade can spin up to 240mph (107m/s).

$v = 107$ m/s
$r = 0.038$ m
$P = 1380$W

Q11: What is the frequency of the blade rotation?

a. 4.48 Hz
b. 10.1 Hz
[✓ c.] 448 Hz
d. 1005 Hz

$$v = \frac{2\pi r}{T} \Rightarrow T = \frac{2\pi r}{v} = \frac{2\pi(0.038)}{(107)}$$

$$T = 2.2314 \times 10^{-3} s$$

$$f = \frac{1}{T} = \frac{1}{2.2314 \times 10^{-3}} = 448.1468 \, Hz$$

Q12: In different blender the blade tip also moves at 107m/s, but the blade length is longer. Is the frequency of rotation larger or smaller? Explain. *(1 mark)*

$$v = \frac{2\pi r}{T} \quad \text{or} \quad v = \left(\frac{2\pi r}{1}\right)\left(\frac{1}{T}\right) \quad \text{where} \quad f = \frac{1}{T}$$

$$v = \left(\frac{2\pi r}{1}\right)\left(\frac{f}{1}\right) \quad \text{or} \quad v = 2\pi \overset{\uparrow}{r} \overset{\downarrow}{f}$$

If speed is same but radius is longer, frequency must be smaller.

KEY

Part 5: Circular Motion (Horizontal Systems)

Q13: A car of mass 1200kg is rounding a horizontal corner of radius 15m. The coefficient of static friction between the road and the car tires is 0.85. What is the maximum speed the car can round the corner at, in m/s?

*(Record your **three-digit** answer in the Numerical Response boxes below)*

1	1	•	2

$m = 1200\,kg$

$r = 15m$

$\mu_s = 0.85$

$F_c = F_f$

$\frac{mv^2}{r} = \mu_s F_N$

$\frac{\cancel{m}v^2}{r} = \mu_s \cancel{m} g$

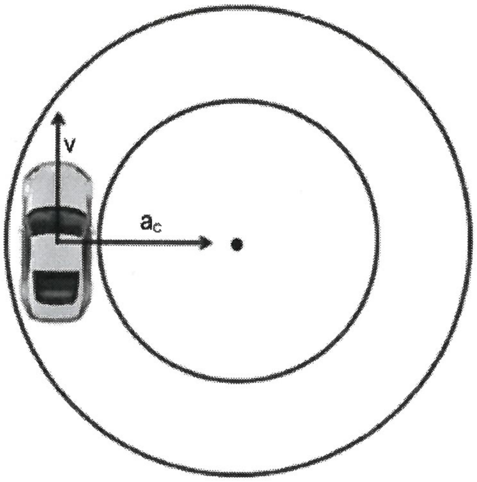
$\frac{v^2}{(15)} = (0.85)(9.81)$

$v^2 = 125.0775$

$v = 11.18\,m/s$

$\boxed{v \approx 11.2\,m/s}$

Q14: A car is travelling in a circle of radius 15m, and has a period of 4.25 seconds. If the car is barely able to maintain circular motion, what is the coefficient of friction? *(2 marks)*



$r = 15m$

$T = 4.25s$

$\mu_s = ?$

$v = \frac{2\pi r}{T} = \frac{2\pi(15)}{(4.25)}$

$v = 22.176\,m/s$

$F_c = F_f$

$\frac{\cancel{m}v^2}{r} = \mu_s \cancel{m} g$

$\frac{(22.176)^2}{(15)} = \mu_s(9.81)$

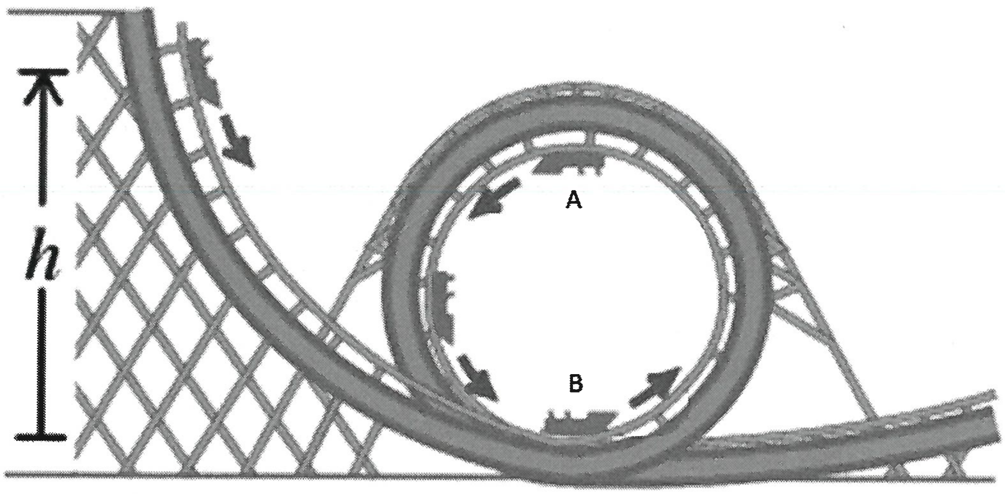
$\boxed{\mu_s = 3.34}$

■ KEY ■

Part 6: Circular Motion (Vertical Systems)

Use the following information to answer Q15-17:

The mass of a rollercoaster car is 2000kg and it turns through a loop of radius 4m at a constant 20m/s.



Q15: The roller coaster is experiencing a centripetal force of ***a.bc*** x 10^{d} N while in the loop, where ***a, b, c,*** and ***d*** are ____, ____, ____, and ____.

*(Record your **four-digit** answer in the Numerical Response boxes below)*

2	0	0	4

m = 200kg
r = 4m
v = 20m/s

$F_c = \frac{mv^2}{r} = \frac{(200)(20)^2}{4}$

$F_c = 20{,}000$ N

$F_c = 2.00 \times 10^4$ N

■ KEY ■

Q16: The normal force acting on the rollercoaster while at the **top** of the loop is ***a.bc*** x 10^{d} N, where ***a, b, c,*** and ***d*** are ___, ___, ___, and ___.

*(Record your **four-digit** answer in the Numerical Response boxes below)*

1	8	0	4

$\underset{+}{\overset{-}{\updownarrow}}$

$$\overset{\downarrow}{F_c} = \overset{\downarrow}{F_g} + \overset{\downarrow}{F_N}$$

$$20{,}000 = (200)(9.81) + F_N$$

$$20{,}000 = 1962 + F_N$$

$$F_N = 18{,}038\ N$$

$$= 1.80 \times 10^{4}\ N$$

Q17: The normal force acting on the rollercoaster while at the **bottom** of the loop is ***a.bc*** x 10^{d} N, where ***a, b, c,*** and ***d*** are ___, ___, ___, and ___.

*(Record your **four-digit** answer in the Numerical Response boxes below)*

2	2	0	4

$\underset{-}{\overset{+}{\updownarrow}}$

$$\overset{\uparrow}{F_c} = \overset{\downarrow}{F_g} + \overset{\uparrow}{F_N}$$

$$20{,}000 = (-1962) + F_N$$

$$F_N = 21{,}962$$

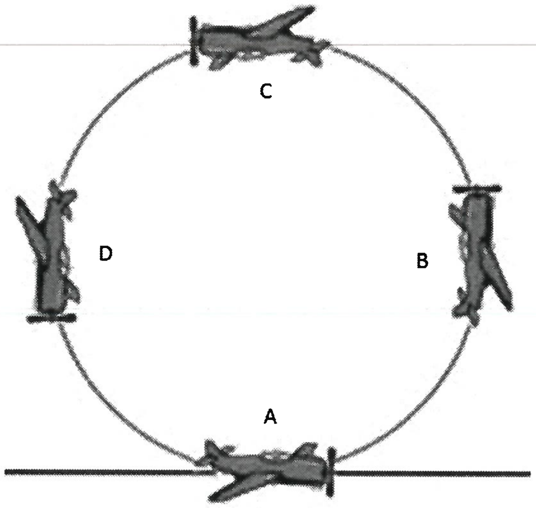
$$= 2.1962 \times 10^{4}\ N$$

$$\approx 2.20 \times 10^{4}\ N$$

■ KEY ■

Use the following information to answer Q18:

A plane of mass 5500kg is making a loop with a diameter of 50m.



Q18: What is the minimum speed the plane needs to make the loop, in m/s?

*(Record your **three-digit** answer in the Numerical Response boxes below)*

1	5	.	7

m = 5500kg

r = 25m

$\overset{\downarrow}{F_c} = \overset{\downarrow}{F_g} + (0N)$

$\frac{\cancel{m}v^2}{r} = \cancel{m}g$

$\frac{v^2}{r} = g$

$v^2 = rg$

$v^2 = (25)(9.81)$

$v^2 = 245.25$

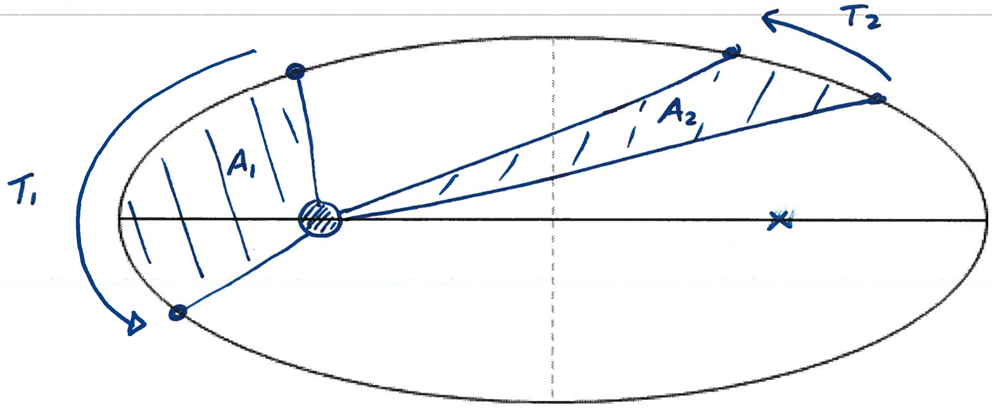
$v = 15.66\ m/s$

$v \approx 15.7\ m/s$

■ KEY ■

Part 7: Kepler's Laws

Q19: Use the diagram of an elliptical orbit and use it to describe Kepler's Laws. *(1 mark)*



① If $A_1 = A_2$, then $T_1 = T_2$

② Elliptical orbit, with one foci being orbited and the other being empty.

③ $\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$

Q20: Neptune is a planet in our solar system with a mean orbital radius of 4.50 x 10^{12} m. Its orbital period is ***a.bc*** x 10^{d} days, where ***a, b, c,*** and ***d*** are ____, ____, ____, and ____.

*(Record your **four-digit** answer in the Numerical Response boxes below)*

Earth's mean orbital radius is 1.50×10^{11} m.

6	0	0	4

Neptune

$r = 4.50\times10^{12}$ m

$T = ?$

Earth

$r = 1.50\times10^{11}$ m

$T = 365$ days

$$\frac{T_N^2}{r_N^3} = \frac{T_E^2}{r_E^3}$$

$$\frac{T_N^2}{(4.50\times10^{12})^3} = \frac{(365)^2}{(1.50\times10^{11})^3}$$

$T_N^2 = 3.597\times10^9$

$T_N = 59{,}975$ days

$\approx 5.9975 \times 10^4$ days

$\approx 6.00\times10^4$ days

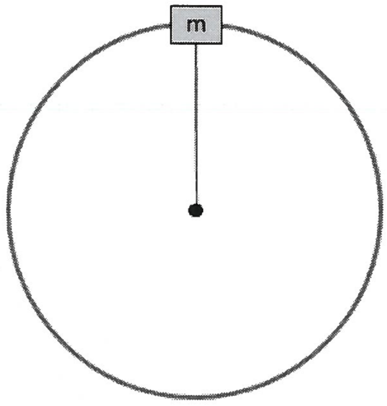
KEY

Part 8: Algebraic and Graphical Circular Motion Questions

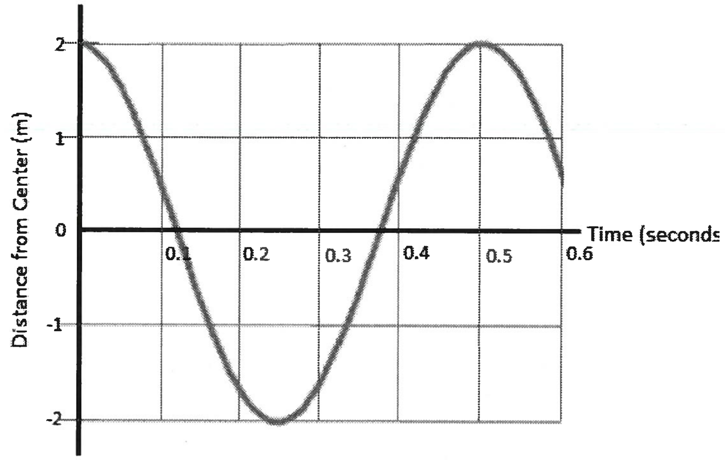
Use the following information to answer Q21-Q24:

A mass of 0.15 kg is swing in a circle with a constant speed. It's height is recorded and graphed as a function of time.

Vertical Circle



Height-Time Graph



Q21: How much time does it take the mass to make 10 revolutions, in seconds?

*(Record your **three-digit** answer in the Numerical Response boxes below)*

5	.	0	0

$m = 0.15$ kg

$r = 2$m (from graph)

$T = 0.5$s (from graph)

If $T = 0.50$ s, then for 10 revolutions, $t = 5.00$s.

Q22: What is the frequency of rotation of the object?

$f = \frac{1}{T} = \frac{1}{0.5} = 2$ Hz.

- a. 0.2 Hz
- b. 0.5 Hz
- [✓ c.] 2.0 Hz
- d. 5.0 Hz

Q23: The object is moving at a constant speed of ***a.bc*** x 10^d m/s, where ***a, b, c,*** and ***d*** are ___, ___, ___, and ___.

*(Record your **four-digit** answer in the Numerical Response boxes below)*

2	5	1	1

$v = \frac{2\pi r}{T} = \frac{2\pi(2)}{0.5} = 25.13$ m/s

$v \approx 2.51 \times 10^1$ m/s

■ KEY ■

Q24: What is the tension in the rope when the mass is at the top of the vertical loop? *(2 marks)*

−

$\updownarrow$

+

$$\overset{\downarrow}{F_c} = \overset{\downarrow}{F_g} + \overset{\downarrow}{F_T}$$

$$\frac{mv^2}{r} = mg + F_T$$

$$\frac{(0.15)(25.13)^2}{2} = (0.15)(9.81) + F_T$$

$$47.3637675 = 1.4715 + F_T$$

$$\boxed{F_T \approx 45.9\ N}$$