

1.08 - Worksheet - Radicals Unit Review**Part 1: Entire to Mixed Radicals**

Q1: For each entire radical, write it as a mixed radical.

$$\sqrt{75x^3y^2}$$

$$\sqrt{3 \cdot 5^2 \cdot x^2 \cdot x \cdot y^2}$$

$$5xy\sqrt{3x}$$

$$\begin{array}{c} 75 \\ \textcircled{3} \wedge \textcircled{25} \\ \textcircled{5} \wedge \textcircled{5} \end{array}$$

$$\sqrt{8x^4y^3z^2}$$

$$\sqrt{2 \cdot 2^2 \cdot x^2 \cdot x^2 \cdot y^2 \cdot y \cdot z^2}$$

$$2x^2yz\sqrt{2y}$$

$$2x^2yz\sqrt{2y}$$

$$\begin{array}{c} 8 \\ \textcircled{2} \wedge \textcircled{4} \\ \textcircled{2} \wedge \textcircled{2} \end{array}$$

Part 2: Non-Permissible Values and RestrictionsQ2: For each of the following radicals, state the *restrictions* on the variable x .

$$\sqrt{2x+1}$$

$$\begin{array}{l} 2x+1 \geq 0 \\ -1 \quad -1 \\ 2x \geq -1 \\ \div 2 \quad \div 2 \\ x \geq -\frac{1}{2} \end{array}$$

$$\sqrt{3x-1}$$

$$\begin{array}{l} 3x-1 \geq 0 \\ +1 \quad +1 \\ 3x \geq 1 \\ \div 3 \quad \div 3 \\ x \geq \frac{1}{3} \end{array}$$

Q3: For each of the following radicals, state the *restrictions* on the variable x .

$$\sqrt{2x+7}$$

$$\begin{array}{l} 2x+7 \geq 0 \\ -7 \quad -7 \\ 2x \geq -7 \\ \div 2 \quad \div 2 \\ x \geq -\frac{7}{2} \end{array}$$

$$\sqrt{5-3x}$$

$$\begin{array}{l} 5-3x \geq 0 \\ -5 \quad -5 \\ -3x \geq -5 \\ \div (-3) \quad \div (-3) \\ x \leq \frac{5}{3} \end{array}$$

Q4: Given the expression $\sqrt{x^2 - 4}$, which of the following correctly describes the *restrictions* on x ?

- a. $x > 2$
- b. $x \geq 2$
- c. $x > 2$ or $x < -2$
- d. $x \geq 2$ or $x \leq -2$**

$$x^2 - 4 \geq 0$$

$$x^2 \geq 4$$

$$\text{so } x \geq 2 \text{ or } x \leq -2$$

Part 3: Adding and Subtracting Radicals

Q5: Simplify radicals and combine like terms.

$$2\sqrt{5} + 7\sqrt{5}$$

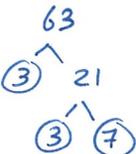
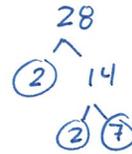
$$9\sqrt{5}$$

$$\sqrt{28} + \sqrt{63}$$

$$\sqrt{2^2 \cdot 7} + \sqrt{3^2 \cdot 7}$$

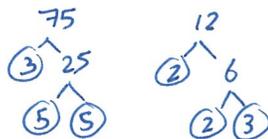
$$2\sqrt{7} + 3\sqrt{7}$$

$$5\sqrt{7}$$



Q6: Simplify radicals and combine like terms.

$$\sqrt{75x} - \sqrt{12x}$$

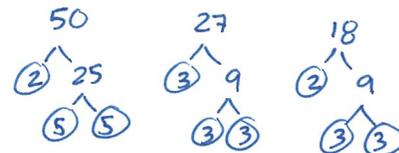


$$\sqrt{3 \cdot 5^2 \cdot x} - \sqrt{2^2 \cdot 3 \cdot x}$$

$$5\sqrt{3x} - 2\sqrt{3x}$$

$$3\sqrt{3x}$$

$$\sqrt{50} - \sqrt{27} + \sqrt{18} - \sqrt{3}$$



$$\sqrt{2 \cdot 5^2} - \sqrt{3 \cdot 3^2} + \sqrt{2 \cdot 3^2} - 1\sqrt{3}$$

$$5\sqrt{2} - 3\sqrt{3} + 3\sqrt{2} - 1\sqrt{3}$$

$$8\sqrt{2} - 4\sqrt{3}$$

Q7: The expression $\sqrt{60} + \sqrt{36} + \sqrt{135}$ simplifies to $a\sqrt{bc} + d$, where a , b , c , and d are ____, ____, ____, and ____.

(Record your **four digit** answer in the Numerical Response boxes below)

5	1	5	6
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$$\sqrt{2^2 \cdot 3 \cdot 5} + \sqrt{2^2 \cdot 3^2} + \sqrt{3^2 \cdot 3 \cdot 5}$$

$$2\sqrt{15} + 2 \cdot 3 + 3\sqrt{15}$$

$$5\sqrt{15} + 6$$

$$a\sqrt{bc} + d$$

Part 4: Rationalizing the Denominator

Q8: For each expression, rationalize the denominator.

$$\frac{\sqrt{5}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{35}}{7}$$

$$\frac{(\sqrt{5}-1)}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{5-\sqrt{5}}{5}$$

Q9 (Math 20-1): For each expression, rationalize the denominator.

$$\frac{(\sqrt{7}-3)(\sqrt{7}-2)}{(\sqrt{7}+2)(\sqrt{7}-2)} = \frac{7-2\sqrt{7}-3\sqrt{7}+6}{7-2\sqrt{7}+2\sqrt{7}-4}$$

$$= \frac{1-5\sqrt{7}}{3}$$

$$\frac{(\sqrt{3}-1)(1-\sqrt{6})}{(1+\sqrt{6})(1-\sqrt{6})} = \frac{\sqrt{3}-\sqrt{18}-1+\sqrt{6}}{1-1\sqrt{6}+1\sqrt{6}-6}$$

$$= \frac{\sqrt{3}-3\sqrt{2}-1+\sqrt{6}}{-5}$$

Q10 (Math 20-1): When rationalizing the denominator of the expression $\frac{\sqrt{5}+3}{\sqrt{5}-1}$, the expression can be simplified to $a + \sqrt{b}$, where a and b are ___ and ___.

(Record your **two digit** answer in the Numerical Response boxes below)

2	5		
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$$\begin{aligned} & \frac{(\sqrt{5}+3)}{(\sqrt{5}-1)} \cdot \frac{(\sqrt{5}+1)}{(\sqrt{5}+1)} \\ &= \frac{5 + 1\sqrt{5} + 3\sqrt{5} + 3}{5 + 1\sqrt{5} - 1\sqrt{5} - 1} = \frac{8 + 4\sqrt{5}}{4} \\ &= 2 + 1\sqrt{5} \\ &= a + \sqrt{b} \end{aligned}$$

Part 5: Multiplying Radicals

Q11: Multiply. Simplify the products where possible.

$$(3\sqrt{2})(4\sqrt{6})$$

$$\begin{aligned} 12\sqrt{12} &= 12\sqrt{2^2 \cdot 3} \\ &= 12(2)\sqrt{3} \\ &= 24\sqrt{3} \end{aligned}$$



$$(\sqrt{5}+1)(3-2\sqrt{2})$$

$$3\sqrt{5} - 2\sqrt{10} + 3 - 2\sqrt{2}$$

Q12: Multiply. Simplify the products where possible.

$$(2\sqrt{x}+5)(3\sqrt{x}-4)$$

$$6x - 8\sqrt{x} + 15\sqrt{x} - 20$$

$$6x + 7\sqrt{x} - 20$$

$$(x\sqrt{x}-5)(4+2\sqrt{x})$$

$$4x\sqrt{x} + 2x(x) - 20 - 10\sqrt{x}$$

$$4x\sqrt{x} + 2x^2 - 20 - 10\sqrt{x}$$

Use the following information to answer Q13-Q15:

An equilateral triangle, $\triangle ABD$, is placed inside a square.

$A_{\text{square}} = 40 \text{ cm}^2$

The area of the square is 40 cm^2 .

Q13 (Math 20-1): The side length of the square, as a mixed radical, is $a\sqrt{bc}$, where a , b , and c are ____, ____, and ____.

(Record your **four digit** answer in the Numerical Response boxes below)

2	1	0	
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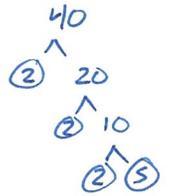
$$x^2 = 40$$

$$x = \sqrt{40}$$

$$x = \sqrt{2^2 \cdot 2 \cdot 5}$$

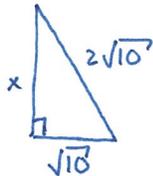
$$x = 2\sqrt{2 \cdot 5}$$

$$x = 2\sqrt{10}$$



Q14 (Math 20-1): The height of the equilateral triangle is

- a. $5\sqrt{2}$
- b. $3\sqrt{10}$
- c. $5\sqrt{10}$
- d. $\sqrt{30}$**



$$a^2 + b^2 = c^2$$

$$x^2 + (\sqrt{10})^2 = (2\sqrt{10})^2$$

$$x^2 + 10 = 4(10)$$

$$x^2 + 10 = 40$$

$$\begin{matrix} -10 & -10 \\ \hline x^2 = 30 \\ x = \sqrt{30} \end{matrix}$$

Q15 (Math 20-1): The area of the equilateral triangle, $\triangle ABD$, is $ab\sqrt{c}$ cm^2 , where a , b , and c are ____, ____, and ____.

(Record your **three digit** answer in the Numerical Response boxes below)

1	0	3	
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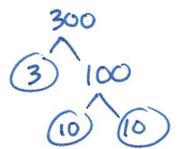
$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}(2\sqrt{10})(\sqrt{30})$$

$$= \sqrt{300}$$

$$= \sqrt{3 \cdot 10^2}$$

$$= 10\sqrt{3}$$



Part 6: Dividing Radicals

Q16: Simplify each expression.

$$\frac{4\sqrt{10}}{6\sqrt{5}} = \frac{4}{6} \sqrt{\frac{10}{5}} = \frac{2}{3} \sqrt{2}$$

$$= \frac{2\sqrt{2}}{3}$$

$$\frac{\sqrt{60}}{\sqrt{20}} = \sqrt{\frac{60}{20}} = \sqrt{3}$$

Q17: Simplify each expression using *two different* methods.

$$\frac{\sqrt{90}}{\sqrt{20}} = \sqrt{\frac{90}{20}} = \sqrt{\frac{9}{2}} = \frac{\sqrt{9}}{\sqrt{2}}$$

$$\frac{\sqrt{9}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{18}}{2} = \frac{\sqrt{2 \cdot 3^2}}{2} = \frac{3\sqrt{2}}{2}$$

$$\frac{\sqrt{90}}{\sqrt{20}} \cdot \frac{\sqrt{20}}{\sqrt{20}} = \frac{\sqrt{1800}}{20} = \frac{\sqrt{2 \cdot 3^2 \cdot 10^2}}{20} = \frac{3 \cdot 10 \sqrt{2}}{20}$$

$$= \frac{3\sqrt{2}}{2}$$

Q18: The expression $\frac{\sqrt{150}}{\sqrt{12}}$ simplifies to $\frac{a\sqrt{b}}{c}$, where *a*, *b*, and *c* are ____, ____, and ____.(Record your *three digit* answer in the Numerical Response boxes below)

5	2	2	
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$$\frac{\sqrt{2 \cdot 3 \cdot 5^2}}{\sqrt{2 \cdot 2 \cdot 3}} = \frac{\sqrt{5^2}}{\sqrt{2}} = \frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$$

$$\frac{a\sqrt{b}}{c}$$

Part 7: Radical Equations (Factorable)

Q19: State the restrictions on x and solve:

$$\sqrt{2x+6} = 7$$

$$2x+6 \geq 0$$

$$\begin{array}{r} -6 \quad -6 \\ 2x \geq -6 \\ \div 2 \quad \div 2 \\ x \geq -3 \end{array}$$

square both sides

$$2x+6 = 49$$

$$\begin{array}{r} -6 \quad -6 \\ 2x = 43 \\ \div 2 \quad \div 2 \\ x = \frac{43}{2} \end{array}$$

$$\sqrt{3x+1} = 4$$

$$3x+1 \geq 0$$

$$\begin{array}{r} -1 \quad -1 \\ 3x \geq -1 \\ \div 3 \quad \div 3 \\ x \geq -\frac{1}{3} \end{array}$$

square both sides

$$3x+1 = 16$$

$$\begin{array}{r} -1 \quad -1 \\ 3x = 15 \\ \div 3 \quad \div 3 \\ x = 5 \end{array}$$

Q20: State the restrictions on x and solve:

$$\sqrt{\frac{5x-2}{3}} = 4$$

$$\frac{5x-2}{3} \geq 0$$

$$\begin{array}{r} \cdot 3 \quad \cdot 3 \\ 5x-2 \geq 0 \\ +2 \quad +2 \\ 5x \geq 2 \\ \div 5 \quad \div 5 \\ x \geq \frac{2}{5} \end{array}$$

square both sides

$$\frac{5x-2}{3} = 16$$

$$\begin{array}{r} \cdot 3 \quad \cdot 3 \\ 5x-2 = 48 \\ +2 \quad +2 \\ 5x = 50 \\ \div 5 \quad \div 5 \\ x = 10 \end{array}$$

$$2\sqrt{5x+1} - 5 = 6$$

$$5x+1 \geq 0$$

$$\begin{array}{r} -1 \quad -1 \\ 5x \geq -1 \\ \div 5 \quad \div 5 \\ x \geq -\frac{1}{5} \end{array}$$

$$2\sqrt{5x+1} = 11$$

$$\begin{array}{r} \div 2 \quad \div 2 \\ \sqrt{5x+1} = \frac{11}{2} \end{array}$$

square both sides

$$5x+1 = \frac{121}{4}$$

$$\begin{array}{r} -1 \quad -1 \\ 5x = \frac{121}{4} - \frac{4}{4} \\ 5x = \frac{117}{4} \\ \div 5 \quad \div 5 \\ x = \frac{117}{20} \end{array}$$

Q21 (Math 20-1): State the restrictions on x and solve:

$$\sqrt{11x+67} = x+7$$

$$11x+67 \geq 0$$

$$\begin{array}{r} -67 \quad -67 \\ 11x \geq -67 \\ \div 11 \quad \div 11 \\ x \geq -\frac{67}{11} \end{array}$$

Square both sides

$$11x+67 = (x+7)(x+7)$$

$$11x+67 = x^2+14x+49$$

$$\begin{array}{r} -11x \quad -67 \quad -11x \quad -67 \\ 0 = x^2+3x-18 \end{array}$$

$$\begin{array}{r} +6 \quad -3 \\ \square + \square = 3 \\ \square \times \square = -18 \end{array}$$

$$0 = (x+6)(x-3)$$

$$\begin{array}{l} \swarrow \quad \searrow \\ x+6=0 \quad x-3=0 \\ -6 \quad -6 \quad +3 \quad +3 \\ x=-6 \quad x=3 \end{array}$$

$$\sqrt{5x+11} = x+3$$

$$5x+11 \geq 0$$

$$\begin{array}{r} -11 \quad -11 \\ 5x \geq -11 \\ \div 5 \quad \div 5 \\ x \geq -\frac{11}{5} \end{array}$$

Square both sides

$$5x+11 = (x+3)(x+3)$$

$$5x+11 = x^2+6x+9$$

$$\begin{array}{r} -5x \quad -11 \quad -5x \quad -11 \\ 0 = x^2+x-2 \end{array}$$

$$\begin{array}{r} +2 \quad -1 \\ \square + \square = 1 \\ \square \times \square = -2 \end{array}$$

$$0 = (x+2)(x-1)$$

$$\begin{array}{l} \swarrow \quad \searrow \\ x+2=0 \quad x-1=0 \\ -2 \quad -2 \quad +1 \quad +1 \\ x=-2 \quad x=1 \end{array}$$

Q22 (Math 20-1): State the restrictions on x and solve:

$$\sqrt{-2x+10} = x-1$$

$$-2x+10 \geq 0$$

$$\begin{array}{r} -10 \quad -10 \\ -2x \geq -10 \\ \div (-2) \quad \div (-2) \\ x \leq 5 \end{array}$$

Square both sides

$$-2x+10 = (x-1)(x-1)$$

$$-2x+10 = x^2-2x+1$$

$$\begin{array}{r} +2x \quad -10 \quad +2x \quad -10 \\ 0 = x^2-9 \end{array}$$

$$\begin{array}{r} +9 \quad +9 \\ 9 = x^2 \\ x=3 \end{array}$$

$$\sqrt{-3x+6} = x-2$$

$$-3x+6 \geq 0$$

$$\begin{array}{r} -6 \quad -6 \\ -3x \geq -6 \\ \div (-3) \quad \div (-3) \\ x \leq 2 \end{array}$$

Square both sides

$$-3x+6 = (x-2)(x-2)$$

$$-3x+6 = x^2-4x+4$$

$$\begin{array}{r} +3x \quad -6 \quad +3x \quad -6 \\ 0 = x^2-x-2 \end{array}$$

$$\begin{array}{r} +1 \quad -2 \\ \square + \square = -1 \\ \square \times \square = -2 \end{array}$$

$$0 = (x+1)(x-2)$$

$$\begin{array}{l} \swarrow \quad \searrow \\ x+1=0 \quad x-2=0 \\ -1 \quad -1 \quad +2 \quad +2 \\ x=-1 \quad x=2 \end{array}$$