KEY

L13 – Vertex Form

Key Ideas

Definitions:

- Vertex Form uses the Vertex (h,k) to quickly build the equation. Similar to “Slope-Point Form” in Math 10C
- Vertex Form $y = a(x-h)^2 + k$ is easier to sketch than Standard Form $y = ax^2 + bx + c$
- In Math 20-1, the vertex is labelled (p,q), making the equation $y = a(x-p)^2 + q$

Part 1 – Vertex Form Transformations

In the equation $f(x) = a(x-h)^2 + k$, what do adjusting ***a, h,*** and ***k*** actually do to the graph?

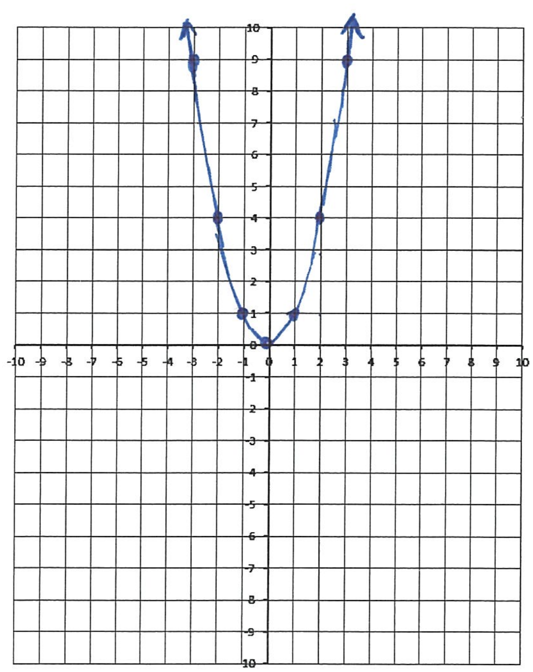
https://phet.colorado.edu/sims/html/graphing-quadratics/latest/graphing-quadratics_en.html

a – Vertical Stretch → Stretch larger or smaller.

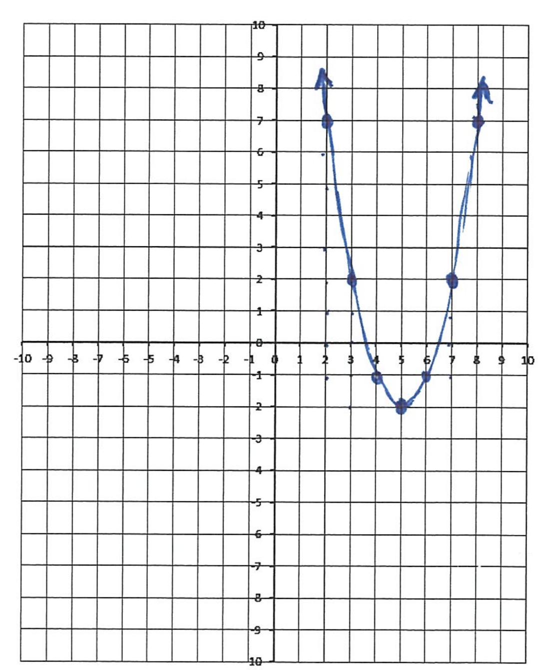
p – Horizontal translation left or right.

q – Vertical translation up or down.

$f(x) = 1(x-0)^2 + 0$



$g(x) = 1(x-5)^2 - 2$



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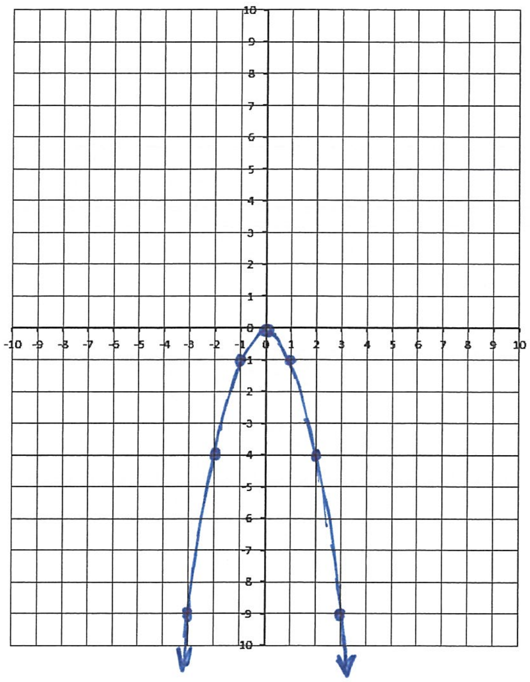
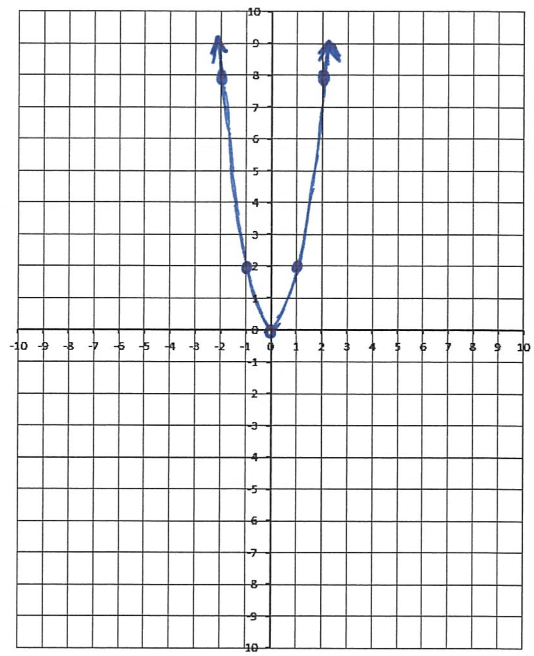
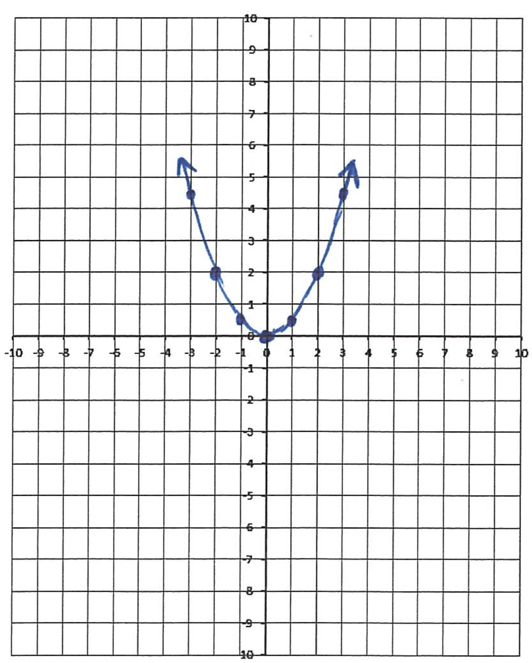
$h(x) = -1(x-0)^2 + 0$



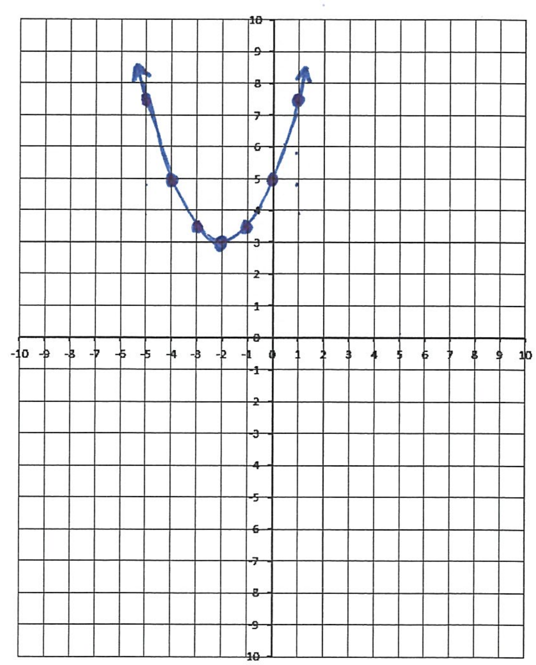
$k(x) = 2(x-0)^2 + 0$



$m(x) = 0.5(x-0)^2 + 0$



$n(x) = 0.5(x+2)^2 + 3$



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Part 2 – Building an Equation using Vertex and a Point

Q1: A parabola has a vertex at (3,5) and passes through the point (1,9). Determine the equation of the line in Vertex Form.

$y = a(x-p)^2 + q$

$y = a(x-3)^2 + 5$ Use (1,9)

$9 = a(1-3)^2 + 5$

$9 = a(-2)^2 + 5$

$9 = a(4) + 5$

$-5 \quad\quad -5$

$4 = a(4)$

$\div 4 \quad\quad \div 4$

$1 = a$

$\boxed{y = 1(x-3)^2 + 5}$

Q2: A parabola has a vertex at (-4,6) and passes throught the point (-2,2). Determine the equation of the line in Vertex Form.

$y = a(x-h)^2 + k$

$y = a(x+4)^2 + 6$ Use (-2,2)

$2 = a(-2+4)^2 + 6$

$2 = a(2)^2 + 6$

$2 = a(4) + 6$

$-6 \quad\quad -6$

$-4 = a(4)$

$\div 4 \quad\quad \div 4$

$-1 = a$

$\boxed{y = -1(x+4)^2 + 6}$

Q3: A parabola has a vertex at (0,6) and passes through the point (3,3). Determine the equation of the line in Vertex Form.

$y = a(x-p)^2 + q$

$y = a(x-0)^2 + 6$ Use (3,3)

$3 = a(3-0)^2 + 6$

$3 = a(9) + 6$

$-6 \quad\quad -6$

$-3 = a(9)$

$\div 9 \quad\quad \div 9$

$-\frac{1}{3} = a$

$\boxed{y = -\frac{1}{3}(x-0)^2 + 6}$

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Part 3 – Converting to Standard Form, $f(x) = ax^2 + bx + c$

Q4: Convert $y = 2(x-3)^2 + 6$ into Standard Form.

$y = 2(x-3)(x-3) + 6$
$y = 2(x^2 - 6x + 9) + 6$
$y = 2x^2 - 12x + 18 + 6$
$y = 2x^2 - 12x + 24$

Q5: Convert $f(x) = \frac{1}{2}(x+2)^2 - 8$ into Standard Form.

$f(x) = \frac{1}{2}(x+2)(x+2) - 8$
$= \frac{1}{2}(x^2 + 4x + 4) - 8$
$= \frac{1}{2}x^2 + 2x + 2 - 8$
$= \frac{1}{2}x^2 + 2x - 6$

Q6: Convert $g(x) = 3(x-1)^2 + 12$ into Standard Form.

$g(x) = 3(x-1)(x-1) + 12$
$= 3(x^2 - 2x + 1) + 12$
$= 3x^2 - 6x + 3 + 12$
$= 3x^2 - 6x + 15$

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Part 4 – Finding the Vertex, Axis of Symmetry, and Zeroes using Vertex Form

Use the following information to answer Q7-Q10:

$$f(x) = 4\left(x + \frac{1}{4}\right)^2 - \frac{81}{4}$$

Q7: Determine the coordinate of the Vertex.

$\left(-\frac{1}{4}, \frac{-81}{4}\right)$

Q8: Determine the equation of the Axis of Symmetry.

$x = -\frac{1}{4}$

Q9: Convert to Standard Form to find the y-Intercept.

$f(x) = 4\left(x+\frac{1}{4}\right)\left(x+\frac{1}{4}\right) - \frac{81}{4}$

$= 4\left(x^2 + \frac{1}{2}x + \frac{1}{16}\right) - \frac{81}{4}$

$= 4x^2 + 2x + \frac{1}{4} - \frac{81}{4}$

$= 4x^2 + 2x - 20$

y-intercept is -20

Q10: Determine the zeroes.

From Vertex Form

$f(x) = 4\left(x+\frac{1}{4}\right)^2 - \frac{81}{4}$

x-int → set $y=0$

$0 = 4\left(x+\frac{1}{4}\right)^2 - \frac{81}{4}$

$+\frac{81}{4} \qquad +\frac{81}{4}$

$\frac{81}{4} = 4\left(x+\frac{1}{4}\right)^2$

$\div 4 \qquad \div 4$

$\frac{81}{16} = \left(x+\frac{1}{4}\right)^2$

$\sqrt{\frac{81}{16}} = \left(x+\frac{1}{4}\right)$

$+\frac{9}{4} = x + \frac{1}{4}$

$-\frac{1}{4} \qquad -\frac{1}{4}$

$\boxed{x = 2}$

$-\frac{9}{4} = x + \frac{1}{4}$

$-\frac{1}{4} \qquad -\frac{1}{4}$

$\boxed{x = -\frac{5}{2}}$

From Standard Form

$f(x) = 4x^2 + 2x - 20$

x-int → set $y=0$

$0 = 4x^2 + 2x - 20$

$\div 2 \quad \div 2 \quad \div 2 \quad \div 2$

$0 = 2x^2 + 1x - 10$

$-4 \quad +5$

$\square + \square = +1$

$\square \times \square = -20$

$0 = 2x^2 - 4x + 5x - 10$

$0 = (2x^2 - 4x) + (5x - 10)$

$0 = 2x(x-2) + 5(x-2)$

$0 = (x-2)(2x+5)$

$x - 2 = 0$

$+2 \quad +2$

$\boxed{x = 2}$

$2x + 5 = 0$

$-5 \quad -5$

$2x = -5$

$\div 2 \quad \div 2$

$\boxed{x = -\frac{5}{2}}$

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Use the following information to answer Q11-Q15:

$$g(x) = 1(x-1)^2 - 9$$

$g(x) = a(x - +1)^2 + -9$

Q11: Determine the coordinate of the Vertex.

$(1, -9)$

Q12: Determine the equation of the Axis of Symmetry.

$x = 1$

Q13: Convert to Standard Form to find the y-Intercept.

$g(x) = 1(x-1)(x-1) - 9$
$= 1(x^2 - 2x + 1) - 9$
$= x^2 - 2x + 1 - 9$
$= x^2 - 2x - 8$
$\downarrow$
y-intercept is -8

Q14: Determine the zeroes.

From Vertex Form	*From Standard Form*
$g(x) = 1(x-1)^2 - 9$ x-int → Set $y = 0$ $0 = 1(x-1)^2 - 9$ $+9 \qquad +9$ $9 = 1(x-1)^2$ $\sqrt{9} = (x-1)$ $+3 = x - 1$; $+1 \quad +1$; $\boxed{x = 4}$ $-3 = x - 1$; $+1 \quad +1$; $\boxed{x = -2}$	$g(x) = x^2 - 2x - 8$ x-int → Set $y = 0$ $0 = x^2 - 2x - 8$ $+2 \quad -4$ $\square + \square = -2$ $\square \times \square = -8$ $0 = (x+2)(x-4)$ $x + 2 = 0$; $-2 \quad -2$; $\boxed{x = -2}$ $x - 4 = 0$; $+4 \quad +4$; $\boxed{x = 4}$

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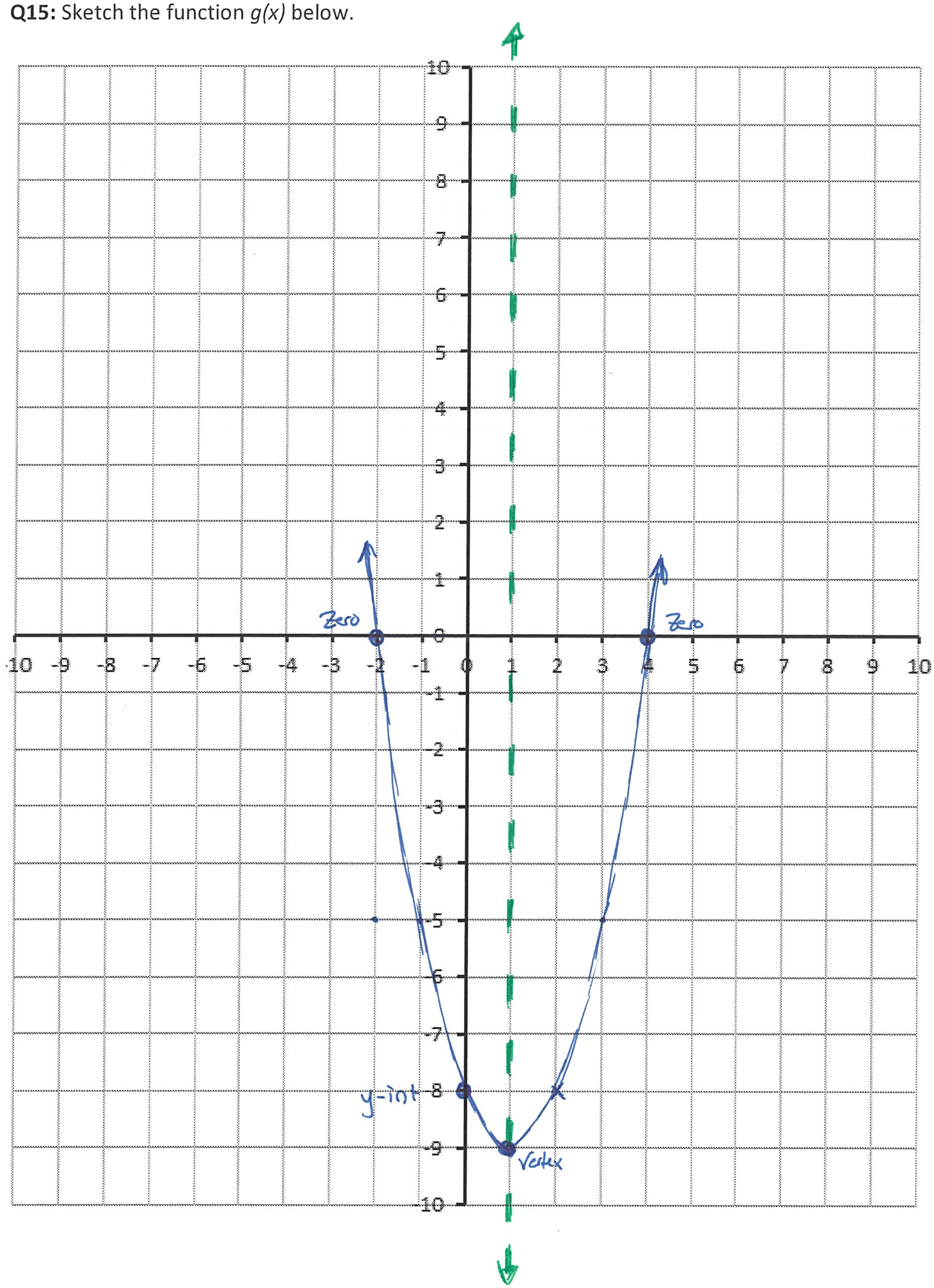
Q15: Sketch the function *g(x)* below.



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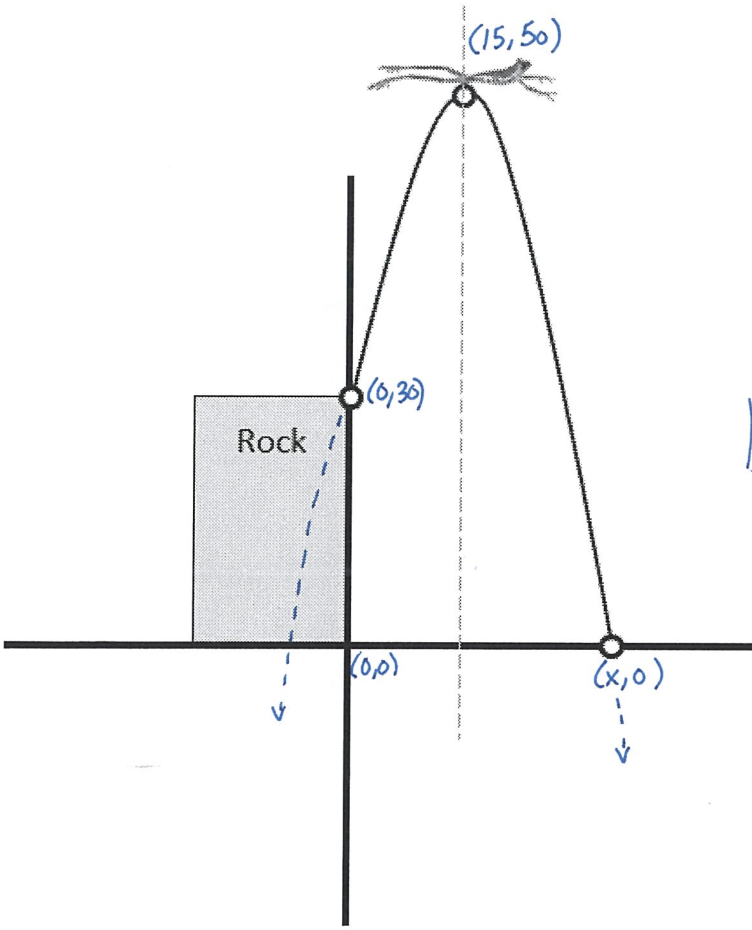
Part 5 – Projectile Motion using Vertex Form

Q16: A frog is standing on a 30cm tall rock. It jumps through the air, reaching a maximum height of 50cm when it is a horizontal distance of 15cm from the rock. How far from the base of the rock does the frog land?



$h(x) = a(x-p)^2 + q$

$h(x) = a(x-15)^2 + 50$

Use 0, 30

$30 = a(0-15)^2 + 50$

$30 = a(225) + 50$

$-50 \qquad -50$

$-20 = a(225)$

$\div 225 \qquad \div 225$

$-\frac{4}{45} = a$

$\boxed{h(x) = -\frac{4}{45}(x-15)^2 + 50}$

Hits bottom at $y = 0$

$0 = -\frac{4}{45}(x-15)^2 + 50$

$-50 \qquad -50$

$-50 = -\frac{4}{45}(x-15)^2$

$\div\left(-\frac{4}{45}\right) \qquad \div\left(-\frac{4}{45}\right)$

$-2250 = -4(x-15)^2$

$\div(-4) \qquad \div(-4)$

$562.5 = (x-15)^2$

$\sqrt{562.5} = x-15$

$+23.72 = x-15$

$+15 \qquad +15$

$\boxed{x = 38.72}$

$-23.72 = x-15$

$+15 \qquad +15$

$\boxed{x = -8.72}$

Two zeroes, but which makes more sense?

$\boxed{x = 38.72 \text{ cm}}$

Q17: State the Domain and Range of the function.

Domain: $\{x \mid 0 \le x \le 38.72, x \in \mathbb{R}\}$

Range: $\{y \mid 0 \le y \le 50, y \in \mathbb{R}\}$