

1.15 - Factored Form**Key Ideas**

Key Ideas:

- If a quadratic equation is factorable, we can easily determine the zeroes of the function.
- We will be representing factored form as  $y = (ax + b)(cx + d)$  rather than the traditional  $y = a(x - r_1)(x - r_2)$  form.
- If we can easily find the zeroes of the function, the axis of symmetry is exactly halfway between them, and the vertex lies on the axis of symmetry.

**Part 1 – Factoring Polynomials**

Q1: Factor the following questions:

$x^2 + 5x + 6$

$$\begin{array}{l} +2 \quad +3 \\ \square + \square = 5 \\ \square \times \square = 6 \end{array}$$

$(x+2)(x+3)$

$$\begin{array}{l} 1,6 \\ 2,3 \end{array}$$

$2x^2 + 7x + 6$

$$\begin{array}{l} +3 \quad +4 \\ \square + \square = 7 \\ \square \times \square = 12 \end{array}$$

$$\begin{array}{l} 2x^2 + 3x + 4x + 6 \\ (2x^2 + 3x) + (4x + 6) \\ \times (2x+3) + 2(2x+3) \\ (2x+3)(x+2) \end{array}$$

$$\begin{array}{l} 1,12 \\ 2,6 \\ 3,4 \end{array}$$

$2x^2 + 11x + 15$

$$\begin{array}{l} +5 \quad +6 \\ \square + \square = 11 \\ \square \times \square = 30 \end{array}$$

$$\begin{array}{l} 2x^2 + 5x + 6x + 15 \\ (2x^2 + 5x) + (6x + 15) \\ \times (2x+5) + 3(2x+5) \\ (2x+5)(x+3) \end{array}$$

$$\begin{array}{l} 1,30 \\ 2,15 \\ 3,10 \\ 5,6 \end{array}$$

**Part 2 – Factoring Quadratics to find the Vertex Coordinates**

Q2: Factor the following quadratics:

$y = x^2 + 5x + 6$

$y = (x+2)(x+3)$

$f(x) = 2x^2 + 7x + 6$

$f(x) = (2x+3)(x+2)$

$y = 2x^2 + 11x + 15$

$y = (2x+5)(x+3)$

Q3: Use the factored form of each quadratic to find the coordinates of the Vertex:

$$y = x^2 + 5x + 6$$

$$y = (x+2)(x+3)$$

Find zeroes

$$0 = (x+2)(x+3)$$

$$\begin{array}{l} \swarrow \quad \searrow \\ x+2=0 \quad x+3=0 \\ -2 \quad -2 \quad -3 \quad -3 \\ x = -2 \quad x = -3 \end{array}$$

Average to find axis of symmetry

$$\frac{(-2) + (-3)}{2} = -2.5 \quad x = -2.5$$

Plug into original eqn

$$y = (-2.5)^2 + 5(-2.5) + 6$$

$$= 6.25 - 12.5 + 6$$

$$= -0.25$$

Vertex at  $(-2.5, -0.25)$

$$f(x) = 2x^2 + 7x + 6$$

$$y = (2x+3)(x+2)$$

Find zeroes

$$0 = (2x+3)(x+2)$$

$$\begin{array}{l} \swarrow \quad \searrow \\ 2x+3=0 \quad x+2=0 \\ x = -1.5 \quad x = -2 \end{array}$$

Average to find axis of sym.

$$\frac{(-1.5) + (-2)}{2} = -1.75 \quad x = -1.75$$

Plug into original eqn

$$y = 2(-1.75)^2 + 7(-1.75) + 6$$

$$= 6.125 - 12.25 + 6$$

$$= -0.125$$

Vertex at  $(-1.75, -0.125)$

$$y = 2x^2 + 11x + 15$$

$$y = (2x+5)(x+3)$$

Find zeroes

$$0 = (2x+5)(x+3)$$

$$\begin{array}{l} \swarrow \quad \searrow \\ 2x+5=0 \quad x+3=0 \\ x = -2.5 \quad x = -3 \end{array}$$

Average to find axis of symmetry

$$\frac{(-2.5) + (-3)}{2} = -2.75 \quad x = -2.75$$

Plug into original eqn.

$$y = 2(-2.75)^2 + 11(-2.75) + 15$$

$$= 15.125 - 30.25 + 15$$

$$= -0.125$$

Vertex at  $(-2.75, -0.125)$

### Part 3 – Putting it all Together

Q4: Given the equation  $f(x) = x^2 + 6x + 8$ , determine (a) the y-intercept, (b) the zeroes, (c) the equation of the axis of symmetry, and (d) the coordinates of the vertex. Finally, (e) sketch the graph on the paper provided.

$$f(x) = x^2 + 6x + 8$$

↳ y-int is 8

$$0 = x^2 + 6x + 8$$

$$0 = (x+2)(x+4)$$

$$\begin{array}{l} \swarrow \quad \searrow \\ x+2=0 \quad x+4=0 \\ x = -2 \quad x = -4 \end{array}$$

Zeroes are  $x = -2$  and  $x = -4$

$$\frac{(-2) + (-4)}{2} = -3 \quad x = -3$$

Axis of symmetry

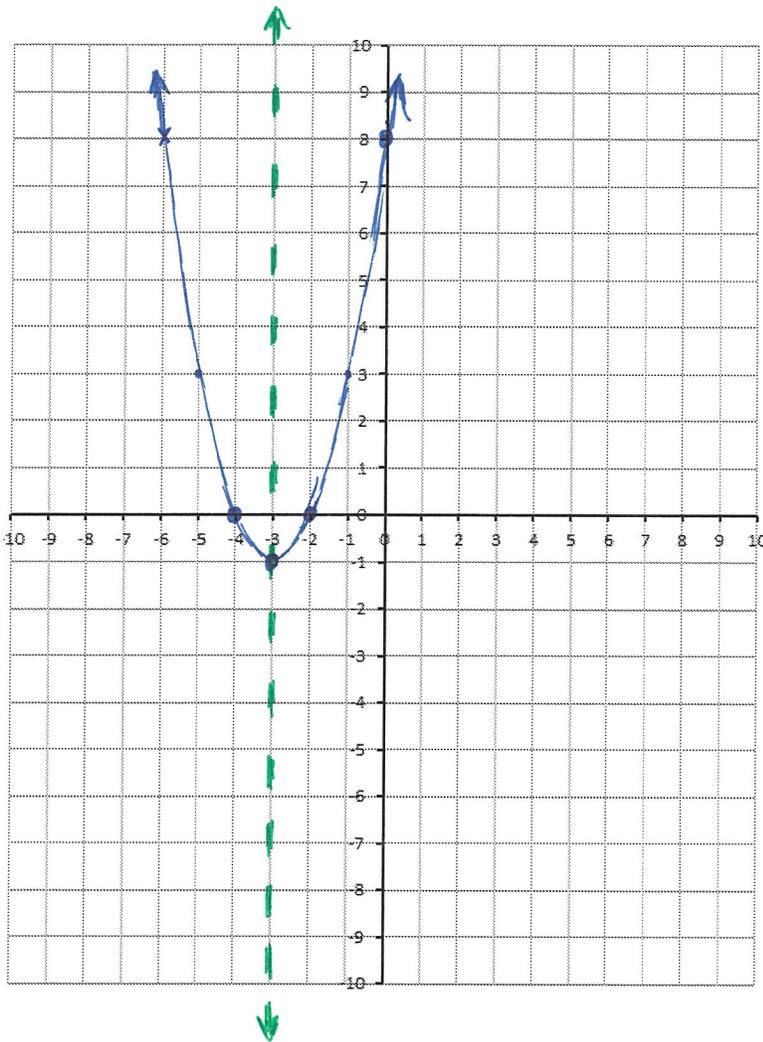
$$f(-3) = x^2 + 6x + 8$$

$$= (-3)^2 + 6(-3) + 8$$

$$= 9 - 18 + 8$$

$$= -1$$

Vertex at  $(-3, -1)$



Q5: Given the equation  $f(x) = 2x^2 - 5x - 3$ , determine (a) the y-intercept, (b) the zeroes, (c) the equation of the axis of symmetry, and (d) the coordinates of the vertex. Finally, (e) sketch the graph on the paper provided

$f(x) = 2x^2 - 5x - 3$

$\begin{matrix} -6 & +1 \\ \square & + \square = -5 \\ \square & \times \square = -6 \end{matrix}$

$\downarrow$

y-int is -3

$0 = 2x^2 - 6x + 1x - 3$   
 $0 = (2x^2 - 6x) + (1x - 3)$   
 $0 = 2x(x-3) + 1(x-3)$   
 $0 = (x-3)(2x+1)$

$\swarrow$                        $\downarrow$

zeroes are  $\boxed{x=3}$  and  $\boxed{x=-\frac{1}{2}}$

$\frac{(3) + (-\frac{1}{2})}{2} = 1.25$        $\boxed{x=1.25}$   
 Axis of symmetry.

$f(1.25) = 2(1.25)^2 - 5(1.25) - 3$   
 $= 3.125 - 6.25 - 3$   
 $= -6.125$

Vertex at  $(1.25, -6.125)$

