KEY

Part 1: Working with Standard and Factored Forms

Use the following information to answer Q1-Q4:

$$f(x) = -x^2 + 4x + 5$$

y-int = + 5

Q1: The y-intercept is

   a. -5
   b. -4
   c. 4
   d. 5

Q2: Converting to *Factored Form*, the zeroes of the function are at *-a* and *+b*, where *a* and *b* are ___ and ___.

         -1   +5

*(Record your **two-digit** answer in the Numerical Response boxes below)*

1	5		

$0 = -x^2 + 4x + 5$

$0 = -1(x^2 - 4x - 5)$

$\div(-1) \quad \div(-1)$

$0 = x^2 - 4x - 5$

$0 = (x-5)(x+1)$

$x - 5 = 0 \qquad x + 1 = 0$

$\boxed{x = 5} \qquad \boxed{x = -1}$

Q3: The axis of symmetry is given by the equation

$$\frac{(5) + (-1)}{2} = 2 \quad \boxed{x = 2}$$

   a. $x = -2$
   b. $x = +2$
   c. $y = -2$
   d. $y = +2$

Q4: The coordinates of the vertex is given by (*a, b*), where *a* and *b* are ___ and ___.

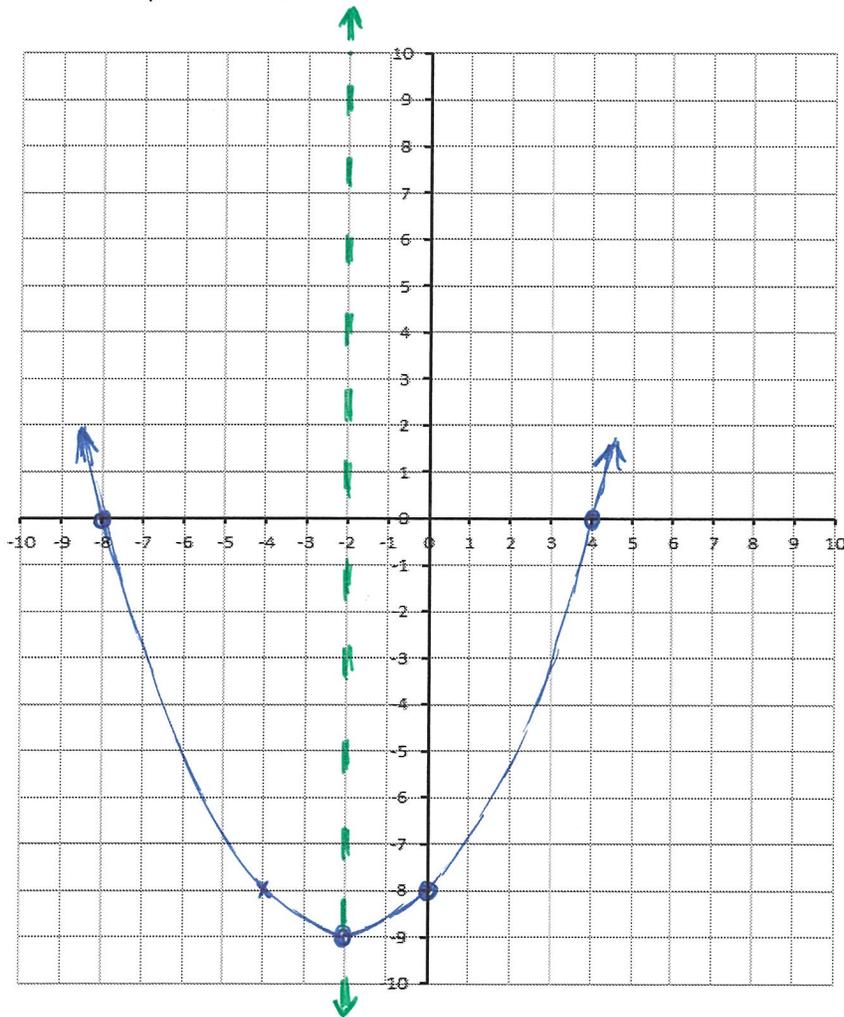*(Record your **two-digit** answer in the Numerical Response boxes below)*

2	9		

$f(2) = -(2)^2 + 4(2) + 5$

$\quad = -4 + 8 + 5$

$\quad = 9$

Vertex is (2, 9)

Q5: A student is working with a quadratic function, and has determined the following information:

- Y-Intercept of -8
- Zeroes of -8 and 4
- Axis of Symmetry is $x = -2$
- Vertex is located at (-2, -9)

Sketch the quadratic function below.

Math 20-2

KEY

Part 2: Working with Vertex Form

Use the following information to answer Q6-Q8:

$$f(x) = \frac{1}{2}(x+4)^2 - 2$$

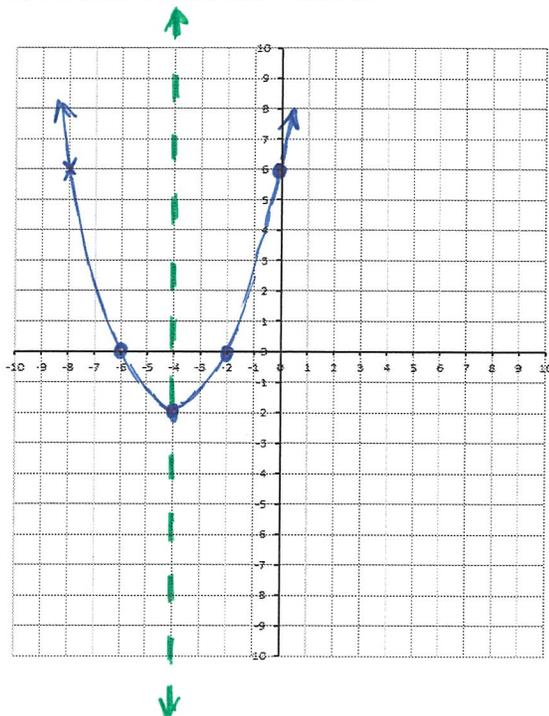**Q6:** Determine (a) the coordinates of the vertex, and (b) the zeroes.

(A) $y = a(x-h)^2 + k$

$y = a(x--4)^2 + -2$

Vertex at $(-4, -2)$

(B) $0 = \frac{1}{2}(x+4)^2 - 2$

$\quad +2 \qquad\qquad\qquad +2$

$2 = \frac{1}{2}(x+4)^2$

$\quad \cdot 2 \quad\; \cdot 2$

$4 = (x+4)^2$

$\sqrt{4} = (x+4)$

$+2 = x+4 \qquad\qquad -2 = x+4$

$\boxed{-2 = x} \qquad\qquad \boxed{-6 = x}$

Q7: Convert the function to *Standard Form* to determine the y-intercept.
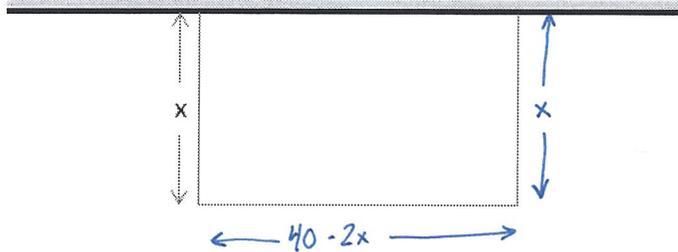
$f(x) = \frac{1}{2}(x+4)(x+4) - 2$

$= \frac{1}{2}(x^2 + 8x + 16) - 2$

$= \frac{1}{2}x^2 + 4x + 8 - 2$

$= \frac{1}{2}x^2 + 4x + 6$

Q8: Sketch the function below.

■ KEY ■

Part 3: Word Problems involving Maximizing Area

Q9: A house owner is building a fence against their barn for chickens to run around. They have 40m of fence. What is the maximum area they can build for their chicken coup?

$A(x) = (L)(w)$
$0 = (x)(40 - 2x)$ ← Already factored form.

$\boxed{x = 0}$

$40 - 2x = 0$
$+2x \quad +2x$
$40 = 2x$
$\div 2 \quad \div 2$
$\boxed{20 = x}$

Axis of symmetry $\dfrac{(0) + (20)}{2} = 10$

$\boxed{x = 10}$

$A(10) = (10)(40 - 2 \cdot 10)$
$= (10)(20)$
$\boxed{= 200 \, m^2}$

Part 4: Word Problems involving Maximizing Profit

Q10: Bob sells tickets to see the documentary "The Politics of the Lizard People". Bob can sell 200 tickets at a cost of $15, but Bob discovered that every time he increases the ticket price by $1, the ticket sales only reduce by 4 people. How much should Bob sell tickets for to maximize profits?

Normal Profit $= (200)(15)$
New Profit $= (200 - 4n)(15 + 1n)$ ← Already factored form.

Let $n =$ number of $1 increases

Find the zeroes.

$0 = (200 - 4n)(15 + 1n)$

$200 - 4n = 0$
$+4n \quad +4n$
$200 = 4n$
$\div 4 \quad \div 4$
$\boxed{n = 50}$

$15 + 1n = 0$
$-15 \quad -15$
$1n = -15$
$\boxed{n = -15}$

Axis of symmetry?
$\dfrac{(50) + (-15)}{2} = 17.5$

$\boxed{x = 17.5}$

Location of Vertex?
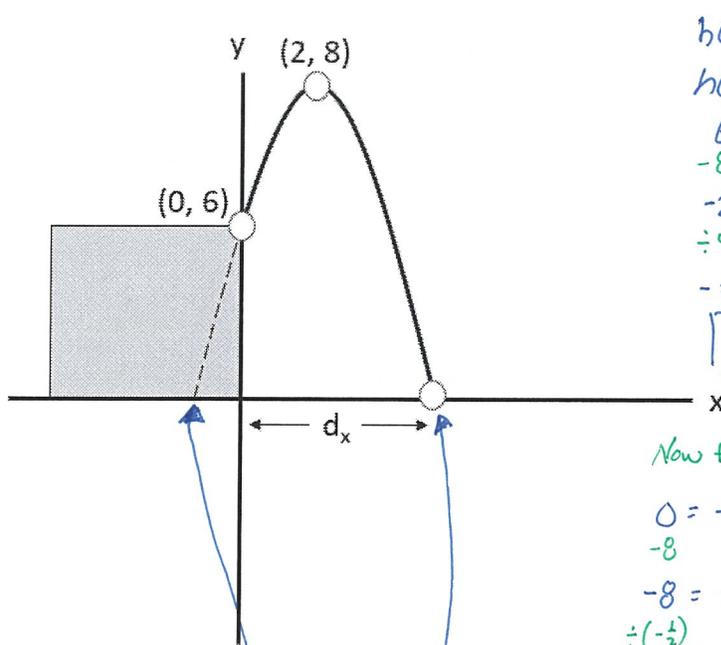$P(n) = (200 - 4[17.5])(15 + 1[17.5])$
$= (130)(32.5)$
$= 4225$

Vertex at $(17.5, 4225)$

So a $17.50 increase

$\boxed{\text{Should charge } \$32.50}$

Part 5: Word Problems involving Projectile Motion

Q11: A frog jumps off a 6cm tall rock, reaching a maximum height of 8cm after travelling a horizontal distance of 2cm. What horizontal distance, d_x , does the frog cover before landing?

$h(x) = a(x-p)^2 + q$　　　Vertex is $(2,8)$

$h(x) = a(x-2)^2 + 8$　　　Use pt $(0,6)$

$\underset{-8}{6} = a(0-2)^2 + \underset{-8}{8}$

$\underset{\div 4}{-2} = \underset{\div 4}{a(4)}$

$-\tfrac{1}{2} = a$

$\boxed{h(x) = -\tfrac{1}{2}(x-2)^2 + 8}$

Now find the zeroes.

$\underset{-8}{0} = -\tfrac{1}{2}(x-2)^2 + \underset{-8}{8}$

$\underset{\div(-\frac{1}{2})}{-8} = \underset{\div(-\frac{1}{2})}{-\tfrac{1}{2}(x-2)^2}$

$16 = (x-2)^2$

$\sqrt{16} = (x-2)$

$\underset{+2\quad+2}{+4 = x-2}$　　　　$\underset{+2\quad+2}{-4 = x-2}$

$\boxed{x=6}$　　　　　　$\boxed{x=-2}$

$\boxed{\text{So Frog travels 6cm}}$