L22 – Solving by Factoring

Key Ideas

1. Quadratic equations can be solved by factoring.
2. The solutions to a quadratic equation are called the roots of the equation.
3. You can factor polynomials in quadratic form.

$$4x^2 + 12x + 9$$

$$(2x + 3)(2x + 3)$$

$$(2x + 3)^2$$

4. You can also factor more complicated expressions using substitution:

$$2(x + 3)^2 - 11(x + 3) + 15 \qquad \text{Let } r = (x + 3)$$

$$2r^2 - 11r + 15$$

$$[2r - 5][r - 3]$$

$$[2(x + 3) - 5][(x + 3) - 3]$$

$$[2x + 6 \quad - 5][x + 3 \quad - 3]$$

$$[2x + 1][x]$$

5. You can factor a difference of squares $P^2 - Q^2$ as $(P + Q)(P - Q)$.

$$\frac{4}{9}x^2 - 16y^2$$

$$\left(\frac{2}{3}x + 4y\right)\left(\frac{2}{3}x - 4y\right)$$

Math 20-2 KEY

Part 1 – Factoring Practice

Q1: Factor the following:

$x^2 + 12x + 35$

+5 +7
□+□ = 12
□×□ = 35

1, 35
5, 7

$(x+5)(x+7)$

$x^2 - 8x + 12$

-2 -6
□+□ = -8
□×□ = 12

1, 12
2, 6
3, 4

$(x-2)(x-6)$

Q2: Factor the following:

$\frac{1}{2}x^2 + 7x + \frac{45}{2}$

+5 +9
□+□ = 14
□×□ = 45

1, 45
3, 15
$\boxed{5, 9}$

$\frac{1}{2}(x^2 + 14x + 45)$

$\frac{1}{2}(x+5)(x+9)$

$-\frac{1}{4}x^2 + \frac{5}{4}x + 6$

-3 +8
□+□ = 5
□×□ = -24

1, 24
2, 12
$\boxed{3, 8}$

$-\frac{1}{4}(x^2 + 5x - 24)$

$-\frac{1}{4}(x-3)(x+8)$

Q3: Factor the following:

$(x-2)^2 + 5(x-2) - 6$

Let $(x-2) = y$

-1 +6
□+□ = 5
□×□ = -6

$y^2 + 5y - 6$

$(y-1)(y+6)$

$(x-2-1)(x-2+6)$

$(x-3)(x+4)$

$\frac{1}{2}(x+3)^2 + \frac{1}{2}(x+3) - 10$

Let $y = x+3$

-4 +5
□+□ = 1
□×□ = -20

1, 20
2, 10
4, 5

$\frac{1}{2}[(x+3)^2 + (x+3) - 20]$

$\frac{1}{2}[y^2 + y - 20]$

$\frac{1}{2}[(y-4)(y+5)]$

$\frac{1}{2}[(x+3-4)(x+3+5)]$

$\frac{1}{2}(x-1)(x+8)$

KEY

Part 2 – Solving Quadratic Equations by Factoring

Q4: Solve for x.

$2x^2 + 5x + 2 = 0$

+1 +4
□+□ = 5
□×□ = 4

1,4
2,2

$2x^2 + 1x + 4x + 2 = 0$

$(2x^2 + 1x) + (4x + 2) = 0$

$x(2x+1) + 2(2x+1) = 0$

$(2x+1)(x+2) = 0$

$2x + 1 = 0$
$-1 \quad -1$
$2x = -1$
$\div 2 \quad \div 2$
$\boxed{x = -1/2}$

$x + 2 = 0$
$-2 \quad -2$
$\boxed{x = -2}$

$2x^2 + 9x + 9 = 0$

+3 +6
□+□ = 9
□×□ = 18

1,18
2,9
3,6

$2x^2 + 3x + 6x + 9 = 0$

$(2x^2 + 3x) + (6x + 9) = 0$

$x(2x+3) + 3(2x+3) = 0$

$(2x+3)(x+3) = 0$

$2x + 3 = 0$
$-3 \quad -3$
$2x = -3$
$\div 2 \quad \div 2$
$\boxed{x = -3/2}$

$x + 3 = 0$
$-3 \quad -3$
$\boxed{x = -3}$

$2x^2 + 12x = -10$
$+10 \quad +10$

$2x^2 + 12x + 10 = 0$

$2(x^2 + 6x + 5) = 0$

+1 +5
□+□ = 6
□×□ = 5

$2(x+1)(x+5) = 0$

$x + 1 = 0$
$-1 \quad -1$
$\boxed{x = -1}$

$x + 5 = 0$
$-5 \quad -5$
$\boxed{x = -5}$

$x^2 = -8x - 15$
$+8x + 15 \quad +8x \quad +15$

$x^2 + 8x + 15 = 0$

+3 +5
□+□ = 8
□×□ = 15

1,15
3,5

$(x+3)(x+5) = 0$

$x + 3 = 0$
$-3 \quad -3$
$\boxed{x = -3}$

$x + 5 = 0$
$-5 \quad -5$
$\boxed{x = -5}$

KEY

Part 3 – Quadratic Equations Word Problems

Use the following information to answer Q5:

In Physics 20, we learn that the height of an object is dependent on the time it is in the air. The equation that governs this relationship is given by

$$\Delta d = v_i t + \frac{1}{2}at^2$$

Where $\Delta d = d_f - d_i$

$$d_f - d_i = v_i t + \frac{1}{2}at^2$$

And we can add d_i to both sides

$$d_f = \frac{1}{2}at^2 + v_i t + d_i$$

Where the acceleration due to gravity is approximately -10 m/s^2. In Math 20-1, we can write this function as follows:

$$h(t) = -5t^2 + v_i t + h_i$$

Where h_i is our initial height, or y-intercept.

KEY

Q5: For a given grasshopper, it is sitting on a 50cm tall ledge when it jumps off with an initial velocity of 15m/s. The equation governing its height, as a function of time, is

$$h(t) = -5t^2 + 15t + 50$$

a. Algebraically determine the zeroes of the function by factoring. What do they represent?

$0 = -5t^2 + 15t + 50$

$\div(-5) \quad \div(-5) \quad \div(-5) \quad \div(-5)$

$0 = t^2 - 3t - 10$

$0 = (t+2)(t-5)$

$+2 \quad -5$

$\square + \square = -3$

$\square \times \square = -10$

$t+2=0 \qquad t-5=0$

Garbage → $\boxed{t=-2}$ $\qquad$ $\boxed{t=5}$ ← Landing time

b. Algebraically determine the coordinates of the vertex, and explain what it represents.

$\frac{(-2)+(5)}{2} = 1.5$

Axis of symmetry at $t=1.5$ sec.

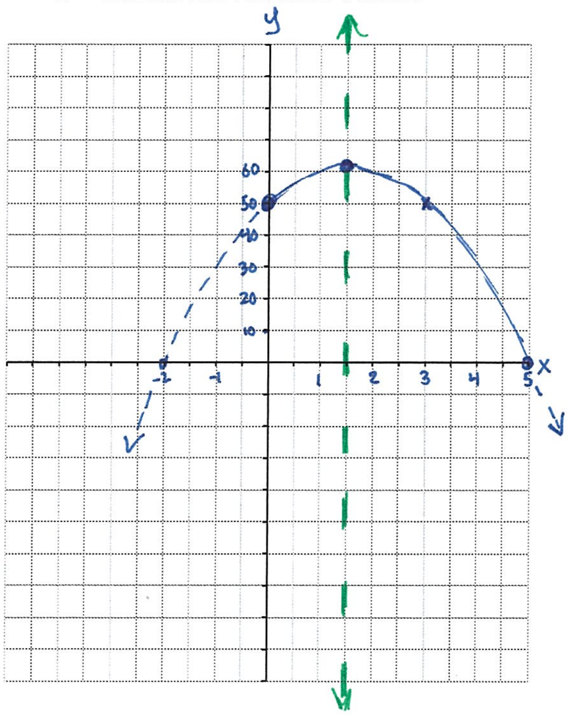
$h(1.5) = -5(1.5)^2 + 15(1.5) + 50$

$h(1.5) = 61.25$

Vertex at $(1.5, 61.25)$

At 1.5 sec, grasshopper is 61.25cm high.

c. Sketch the function below.



d. State the Domain and Range of the function.

Domain: $\{t \mid 0 \leq t \leq 5, t \in \mathbb{R}\}$

Range: $\{h \mid 0 \leq h \leq 61.25, h \in \mathbb{R}\}$