

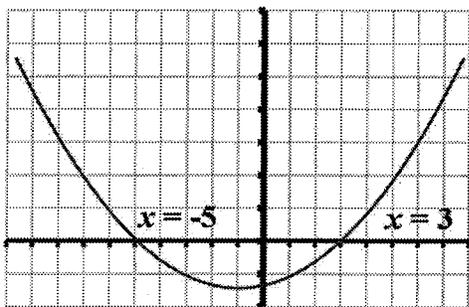
2.4 - Solving by Quadratic Equation

Key Ideas

Solving a quadratic equation

$$x^2 + 2x - 15 = 0$$

Method 1: Graph



Method 2: Factor

$$(x-3)(x+5) = 0$$

Method 3: Quadratic Formula

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

While in Standard Form, $y = ax^2 + bx + c$, the zeroes of the function can be solved using the Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This gives you two possible solutions:

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Use this to solve for the x-intercepts of a quadratic function that cannot be factored easily.

Part 1 – Math 20-1: The Derivation of the Quadratic Formula

Step #1: Switch to Vertex Form

$$y = ax^2 + bx + c$$

$$y = (ax^2 + bx) + c$$

$$y = a\left(x^2 + \frac{b}{a}x\right) + c$$

$$y = a\left(x^2 + \frac{b}{2a}x + \frac{b}{2a}x\right) + c$$

$$y = a\left(x^2 + \frac{b}{2a}x + \frac{b}{2a}x + \frac{b^2}{4a^2}\right) + c - \frac{b^2}{4a}$$

$$y = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

Step #2: Solve for the Zeroes

$$0 = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

$$\frac{b^2}{4a} - c = a\left(x + \frac{b}{2a}\right)^2$$

$$\frac{b^2}{4a} - \frac{4ac}{4a} = a\left(x + \frac{b}{2a}\right)^2$$

$$\frac{b^2 - 4ac}{4a} = a\left(x + \frac{b}{2a}\right)^2$$

$$\frac{b^2 - 4ac}{4a^2} = \left(x + \frac{b}{2a}\right)^2$$

$$\pm \sqrt{\frac{b^2 - 4ac}{4a^2}} = x + \frac{b}{2a}$$

$$\pm \frac{\sqrt{b^2 - 4ac}}{2a} = x + \frac{b}{2a}$$

$$-\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = x$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = x$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Part 2 – When would we use it?

This is ideally used for a quadratic that isn't factorable.

Q1: Solve $x^2 + 13x + 40 = 0$ using each method.

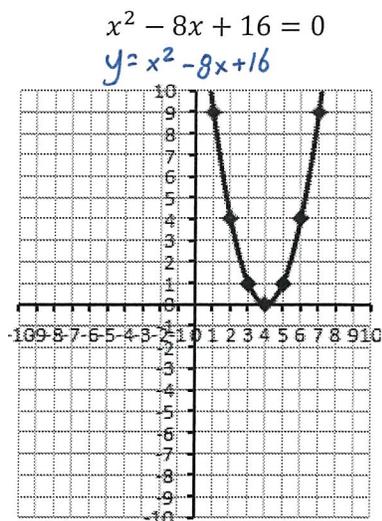
Factoring	Quadratic Formula
$x^2 + 13x + 40 = 0$ $(x+5)(x+8) = 0$ <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> $x+5=0$ $x=-5$ </div> <div style="text-align: center;"> $x+8=0$ $x=-8$ </div> </div>	$x^2 + 13x + 40 = 0$ $a=1 \quad b=13 \quad c=40$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(13) \pm \sqrt{13^2 - 4(1)(40)}}{2(1)}$ $x = \frac{-13 \pm \sqrt{169 - 160}}{2}$ $x = \frac{-13 \pm \sqrt{9}}{2}$ $x = \frac{-13 \pm 3}{2}$ <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> $x = \frac{-13+3}{2}$ $x = -5$ </div> <div style="text-align: center;"> $x = \frac{-13-3}{2}$ $x = -8$ </div> </div>

Q2: Solve $x^2 + 4x - 4 = 0$ using each method

Factoring	Quadratic Formula
$x^2 + 4x - 4 = 0$ <div style="display: flex; justify-content: space-around; align-items: center; margin-top: 10px;"> <div style="text-align: center;"> $\square + \square = 4$ $\square \times \square = 4$ </div> </div> <p style="text-align: center; margin-top: 10px;"><u>Not factorable!</u></p>	$x^2 + 4x - 4 = 0$ $a=1 \quad b=4 \quad c=-4$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-4)}}{2(1)}$ $x = \frac{-4 \pm \sqrt{32}}{2}$ $x = \frac{-4 \pm 4\sqrt{2}}{2}$ <div style="display: flex; justify-content: space-around; align-items: center; margin-top: 10px;"> <div style="text-align: center; border: 1px solid black; padding: 5px;"> $x_1 = -2 + 2\sqrt{2}$ ≈ 0.83 </div> <div style="text-align: center; border: 1px solid black; padding: 5px;"> $x_2 = -2 - 2\sqrt{2}$ ≈ -4.83 </div> </div>

Part 3 – Two, One, or Zero Answers

Q3: For each of the following functions, solve for the zeroes.



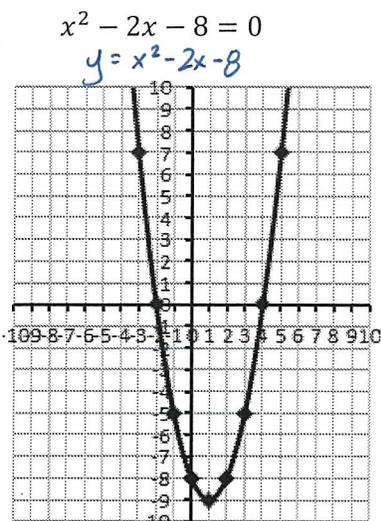
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{64 - 4(1)(16)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{0}}{2}$$

$$x = \frac{8}{2}$$

$$\boxed{x = 4}$$



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{4 - 4(1)(-8)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 + 32}}{2}$$

$$x = \frac{2 \pm \sqrt{36}}{2}$$

$$x = \frac{2 \pm 6}{2}$$

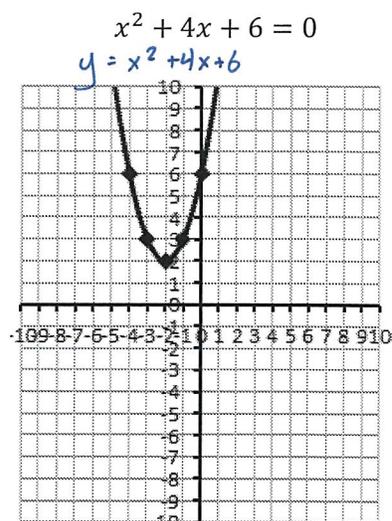
↙ ↘

$$x = \frac{2+6}{2}$$

$$x = \frac{2-6}{2}$$

$$\boxed{x_1 = 4}$$

$$\boxed{x = -2}$$



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{16 - 4(1)(6)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 - 24}}{2}$$

$$x = \frac{-4 \pm \sqrt{-8}}{2} \quad \&$$

No solutions

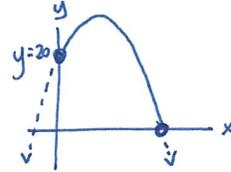
Part 4 – Examples from Word Problems

Use the following information to answer Q4:

An object is shot vertically off the edge of a 20m tall cliff at 4m/s [up]. The height of the object, as a function of time, is given by the equation

$$h(t) = -5t^2 + 4t + 20$$

where $h(t)$ is measured in meters, and t is measured in seconds.



Q4: Use the quadratic equation to determine how long it takes the object to reach the ground.

$$y = -5x^2 + 4x + 20$$

$$a = -5 \quad b = 4 \quad c = 20$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{16 - 4(-5)(20)}}{2(-5)}$$

$$x = \frac{-4 \pm \sqrt{416}}{-10}$$

$$x_1 = \frac{-4 + \sqrt{416}}{-10}$$

$$x_2 = \frac{-4 - \sqrt{416}}{-10}$$

$$x_1 = -1.64$$

$$x_2 = +2.44$$

Takes 2.44 sec to hit the ground