KEY

Math 20-2

L31 – Inductive Reasoning

Terminology

Conjecture: A testable expression that is based on available evidence but is not yet proved.

Inductive Reasoning: Drawing a general conclusion by observing patterns and identifying properties in specific examples.

Counter Example: An example that invalidates a conjecture.

Topic #1: Inductive Reasoning – Linear Relations

Q1: Georgia, a fabric artist, has been patterning with equilateral triangles. Georgia believes that the 10th figure in the pattern will have 100 upright triangles. Do you agree? Explain.

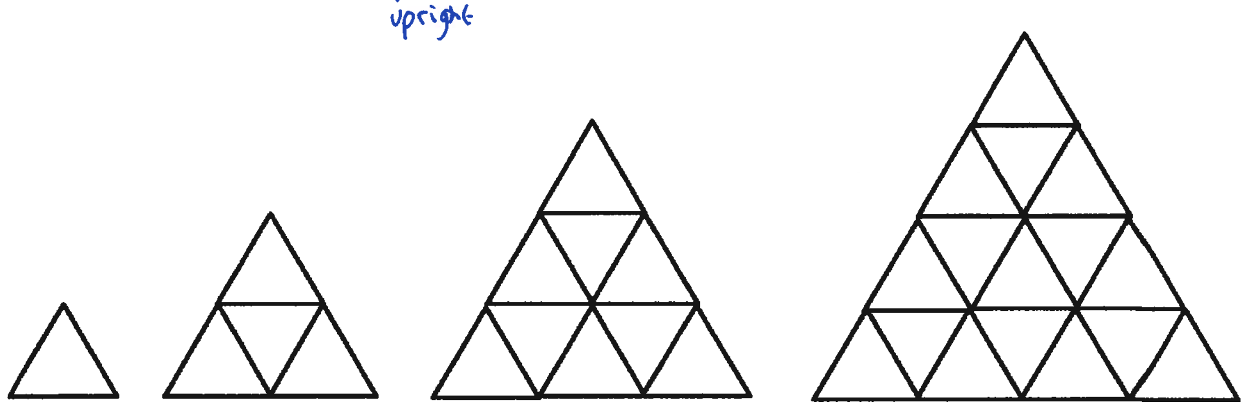

Figure	Fig 1	Fig 2	Fig 3	Fig 4
Base 1 Triangles	1	3	6	10
Base 2 Triangles	0	1	3	6
Base 3 Triangles	0	0	1	3
Base 4 Triangles	0	0	0	1
Total Triangles	1	4	10	20

The pattern for any row is $1 \xrightarrow{+2} 3 \xrightarrow{+3} 6 \xrightarrow{+4} 10 \xrightarrow{+5} 15 \xrightarrow{+6} 21 \xrightarrow{+7} 28 \xrightarrow{+8} 36 \xrightarrow{+9} 45 \xrightarrow{+10} 55$

For a base "n" triangle, the total upright triangles is the sum of the first "n" numbers.

So the 10th triangle should have $1+3+6+\ldots+55$ triangles, or 220 triangles.

Note: The following appears to work, but doesn't accurately predict the 5th figure.

$1\times 0+1 = 1$

$2\times 1+2 = 4$

$3\times 2+4 = 10$

$4\times 3+8 = 20$

$(n)(n-1)+2^{n-1}$

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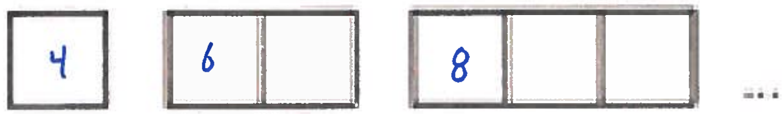
Q2: Bob likes blocks. Bob believes that the perimeter of the 10th set of blocks will be an even number. Do you agree? Explain.



$$\left.\begin{aligned} P &= 2n+2 \\ &= 2(n+1) \end{aligned}\right\}$$ Deductive Reasoning.

Yeah, it's even.

Q3: A series of triangles can be made using toothpicks, per the diagrams below. Make a conjecture about the next 10 diagrams. Is this testable? Do any counter-examples exist?

$3 \xrightarrow{+2} 5 \xrightarrow{+2} 7$

Conjecture: Add 2 toothpicks every time.

or $N = 2n+1$

10th diagram → 21 toothpicks

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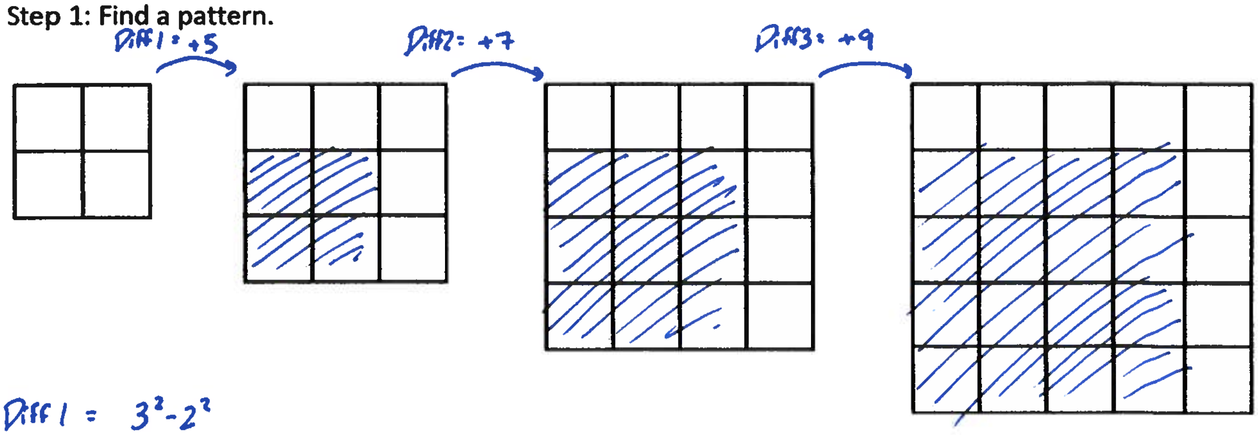
Topic#2: Inductive Reasoning – Non-Linear Patterns

Q4: Make a conjecture about the difference between two consecutive perfect squares.

Step 1: Find a pattern.



$\text{Diff } 1 = 3^2 - 2^2$

$\text{Diff } 2 = 4^2 - 3^2$

$\text{Diff } 3 = 5^2 - 4^2$

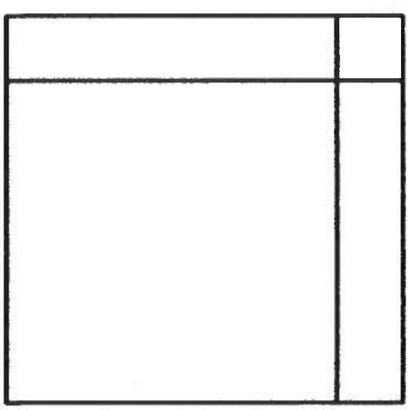
$$
\begin{aligned}
\text{Diff } n &= (n+2)^2 - (n+1)^2 \\
&= (n^2 + 4n + 4) - (n^2 + 2n + 1) \\
&= 2n + 3
\end{aligned}
$$

Step 2: Determine your conjecture. Is it testable? Do counter examples exist?

Yes, it is testable.

Step 3: If possible, generalize the pattern.

$\text{Diff } n = 2n + 3$



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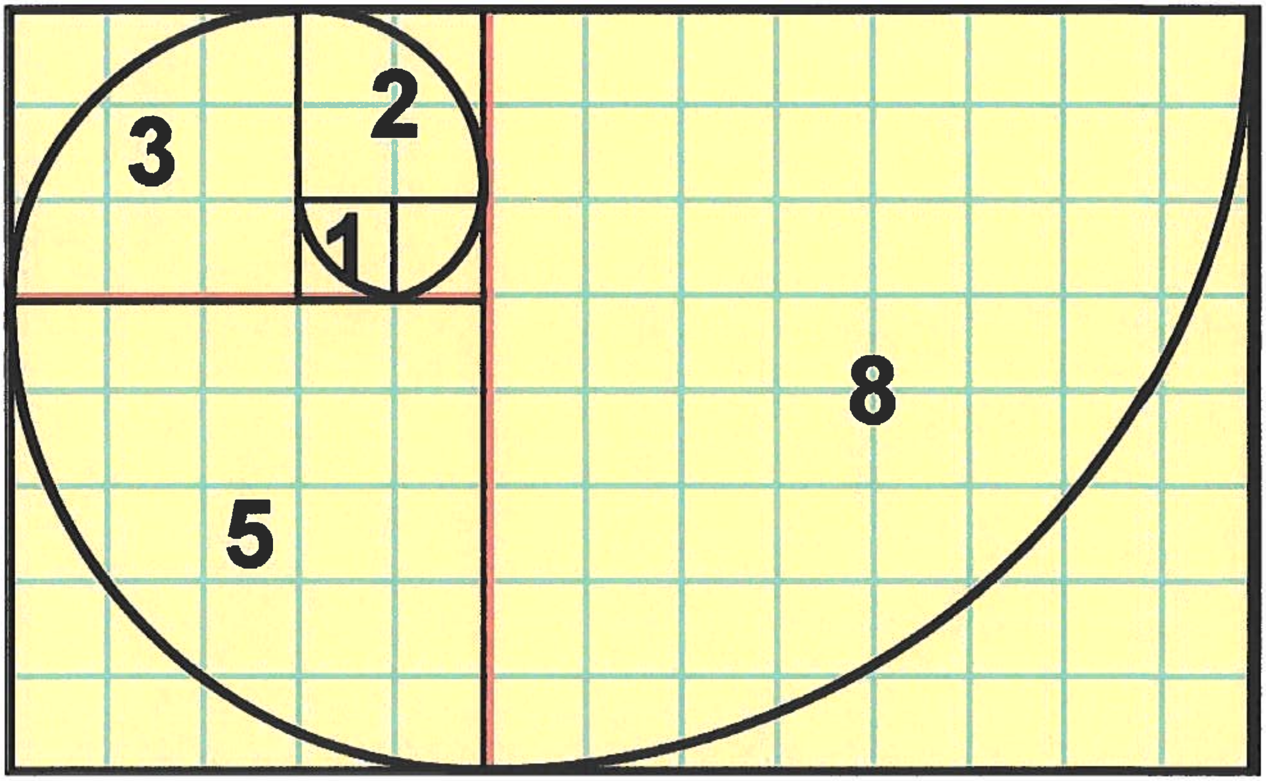
Q5: Make a conjecture about the pattern below, and determine the next 5 numbers.



1, 2, 3, 5, 8, 13, 21, ...

Every number is the sum of the two previous numbers.