

32 - Deductive Reasoning

Terminology

Proof: A mathematical argument showing that a statement is valid in all cases, or that no counterexample exists.

Generalization: A principle, statement, or idea that has general application.

Deductive Reasoning: Drawing a specific conclusion through logical reasoning by starting with general assumptions that are known to be valid.

Inductive vs. Deductive Reasoning

INDUCTIVE reasoning

VS

DEDUCTIVE reasoning

No matter how unrealistic that sounds, in many fields, such as science and law, "proof" simply doesn't exist; there can only be facts and evidence that lead you to certain conclusions.

INDUCTIVE REASONING

- Someone who uses INDUCTIVE reasoning makes **specific observations** and **then draws a general conclusion**.
- When you're using inductive reasoning, correct observations won't necessarily lead you to a correct general conclusion.

EXAMPLES

1. Every quiz has been easy. Therefore, the test will be easy.
2. The teacher used PPT in the last few classes. Therefore, the teacher will use PPT tomorrow.



DEDUCTIVE REASONING

- DEDUCTIVE reasoning is a **specific conclusion** follows a **general theory**.
- When you're using deductive reasoning, your conclusion will be correct if all the statements you say is correct.

EXAMPLES

1. All students in this class play guitar.
Sam is a student of this class.
--> Therefore, Sam plays guitar.
2. At the conference, all the people present are thirty or older.
Maria is in the room.
--> Therefore, Maria is at least thirty.

Expressing Even Numbers, Odd Numbers, and Consecutive Numbers

How do you express a number? x or y

How do you express an even number? $2x$ or $2y$

How do you express an odd number? $2x+1$ or $2y+1$

How do you express consecutive numbers? $x, x+1, x+2, \dots$

Topic #3: Deductive Reasoning – Even/Odd and Consecutive Numbers

Q1: Prove that the sum of two odd numbers is an even number.

Let $2x+1$ and $2y+1$ be odd numbers.

$(2x+1) + (2y+1)$	Add numbers
$2x+2y+2$	Simplify
$2(x+y+1)$	Divisible by 2. Must be even

Q2: Prove that the sum of two even numbers is an even number.

Use $2x$ and $2y$	Both are even.
$2x + 2y$	Add
$2(x+y)$	Divisible by 2. Must be even.

Q3: Prove that the sum of an even and an odd number is an odd number.

Use $2x$ and $2y+1$	Even ($2x$) and Odd ($2y+1$)
$(2x) + (2y+1)$	Add numbers.
$2x+2y+1$	Simplify
$(2x+2y)+1$	Group first 2.
$2(x+y)+1$	Even +1, so odd.

Q4: Prove that the product of an even and odd number is an even number.

Use $2x$ and $2y+1$	Even ($2x$) and Odd ($2y+1$)
$(2x)(2y+1)$	Multiply
$4xy + 2x$	Simplify
$2(2xy+1)$	Divisible by 2. Must be even.

Q5: Prove that the product of two odd numbers is an odd number.

Use $2x+1$ and $2y+1$	Odd ($2x+1$) and odd ($2y+1$)
$(2x+1)(2y+1)$	Multiply
$4xy + 2x + 2y + 1$	Simplify
$(4xy + 2x + 2y) + 1$	Group
$2(2xy + x + y) + 1$	Even +1, so odd.

Topic #3: Deductive Reasoning - Physics

Q6: The force of gravity, F , between two objects that are distance r apart can be modeled by the equation:

$$F = \frac{100}{(r)^2}$$

Prove that when the distance is tripled, the force is $1/9^{\text{th}}$ of the original amount. (2 marks)

Math	Explanation
$F = \frac{100}{(3r)^2}$	Triple distance.
$F = \frac{100}{9r^2}$	$(ab)^n = a^n b^n$, so $(3r)^2 = 9r^2$
$F = \frac{1}{9} \left(\frac{100}{r^2} \right)$	Factor out $1/9^{\text{th}}$. So $1/9^{\text{th}}$ of original amount.

Q7: The Kinetic Energy, E , of a particular object depends upon its mass, m , and its speed, v , per the equation below:

$$E = \frac{1}{2} m(v)^2$$

Prove that when the speed is halved, the kinetic energy is one quarter of the original amount. (2 marks)

Math	Explanation
$E = \frac{1}{2} m \left(\frac{1}{2} v \right)^2$	Half the speed.
$E = \frac{1}{2} m \left(\frac{1}{4} v^2 \right)$	$(ab)^n = a^n b^n$, so $\frac{1}{2} v^2 = \frac{1}{4} v^2$
$E = \frac{1}{4} \left(\frac{1}{2} m v^2 \right)$	Bring $1/4$ out in front. So $1/4$ of original amount.

Topic #4: Deductive Reasoning – Invalid Proofs

Q8: Hossai is trying to prove the following number trick:

Choose any number. Add 3. Double it. Add 4. Divide by 2. Take away the number you started with.

Each time Hossai tries the trick, she ends up with 5. Her proof, however, does not give the same result.

Hossai's Proof (as a "2-Column Proof")

n	Choose any number
$n + 3$	Add 3
$2n + 6$	Double it
$2n + 10$	Add 4
$2n + 5$	Divide by 2
$n + 5$	Take away the number you started with

Where is the error in Hossai's proof?

When you divide $2n+10$ by 2, you get $n+5$

Q9: Mr. Bayer believes that he just proved that $2 = 1$.

Bayer's Proof (as a "2-Column Proof")

Let $a = b$	Set $a = b$
$a^2 = ab$	Multiply both sides by a
$a^2 - b^2 = ab - b^2$	Subtract b^2 from both sides
$(a + b)(a - b) = b(a - b)$	Factor each side
$a + b = b$	Divide both sides by $(a-b)$
$b + b = b$	Replace a with b , since $a = b$
$2b = b$	Collect like terms on left side
$2 = 1$	Divide both sides by b

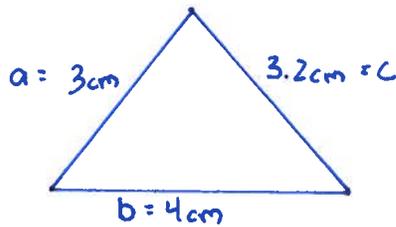
Where is the error in Bayer's proof?

If $a = b$ then dividing by $(a-b)$ is the same as dividing by zero.

Q10: Mr. Bayer claims that the cube of a number is always larger than the number itself. Give one counter-example to this claim. (1 mark)

$$0.5^3 = 0.125, \text{ which is smaller than } 0.5$$

Q11: Mr. Bayer makes the claim that Pythagoras Theorem ($a^2 + b^2 = c^2$) works for every triangle. Using a ruler, create one counter-example where this is not true. Label the lengths of each side. (1 mark)



Only works for 90° triangles.

Q12: Mr. Bayer decides to show that the product of an odd number and 3 is always an odd number. However, he made a mistake somewhere. Find which line has the mistake, and correct his proof. (3 marks)

Line	Math	Explanation
1	Let $(2x+1)$ be my odd number.	Even number plus one, which makes it odd.
2	$(2x+1)(3)$	Multiply my numbers
3	$6x+4$	Simplify
4	$2(3x+2)$	Hold on... this is divisible by 2. Did I make a mistake?

Line	Math	Explanation
1	Let $(2x+1)$ be my odd number.	Even number plus one, which makes it odd.
2	$(2x+1)(3)$	Multiply my numbers.
3	$6x+3$	Simplify.
4	$(6x+2) + 1$	Split and group.
5	$2(3x+1) + 1$	Even number + 1 is odd.