Math 20-2

KEY

L32 – Deductive Reasoning

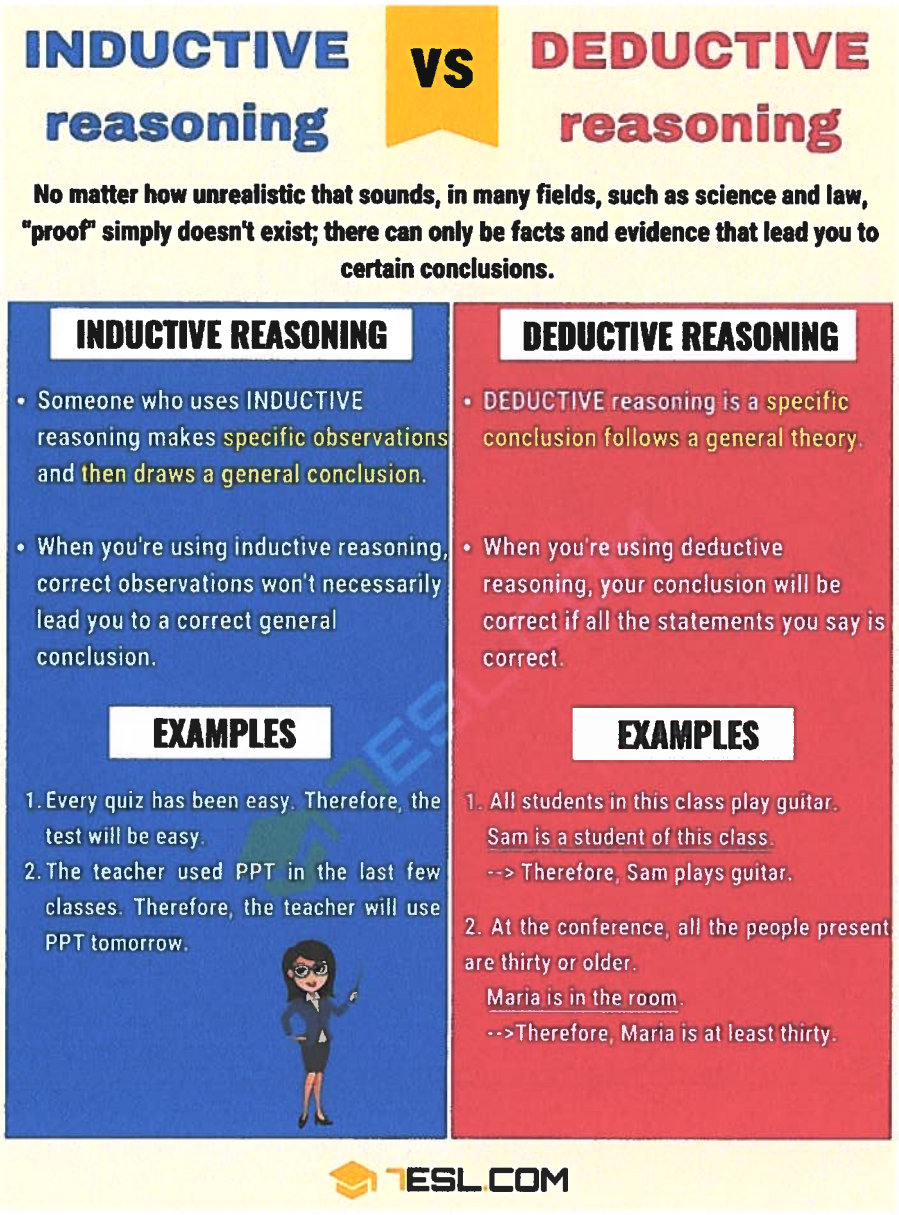
Terminology

Proof: A mathematical argument showing that a statement is valid in all cases, or that no counterexample exists.

Generalization: A principle, statement, or idea that has general application.

Deductive Reasoning: Drawing a specific conclusion through logical reasoning by starting with general assumptions that are known to be valid.

Inductive vs. Deductive Reasoning



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Expressing Even Numbers, Odd Numbers, and Consecutive Numbers

How do you express a number? x or y

How do you express an even number? $2x$ or $2y$

How do you express an odd number? $2x+1$ or $2y+1$

How do you express consecutive numbers? $x,\ x+1,\ x+2,\ \ldots$

Topic #3: Deductive Reasoning – Even/Odd and Consecutive Numbers

Q1: Prove that the sum of two odd numbers is an even number.

Let $2x+1$ and $2y+1$ be odd numbers.

$(2x+1)+(2y+1)$	Add numbers
$2x+2y+2$	Simplify
$2(x+y+1)$	Divisible by 2. Must be even

Q2: Prove that the sum of two even numbers is an even number.

Use $2x$ and $2y$	Both are even.
$2x+2y$	Add
$2(x+y)$	Divisible by 2. Must be even.

Q3: Prove that the sum of an even and an odd number is an odd number.

Use $2x$ and $2y+1$	Even $(2x)$ and Odd $(2y+1)$
$(2x)+(2y+1)$	Add numbers.
$2x+2y+1$	Simplify
$(2x+2y)+1$	Group first 2.
$2(x+y)+1$	Even +1, so odd.

Q4: Prove that the product of an even and odd number is an even number.

Use $2x$ and $2y+1$	Even $(2x)$ and Odd $(2y+1)$
$(2x)(2y+1)$	Multiply
$4xy+2x$	Simplify
$2(2xy+1)$	Divisible by 2. Must be even.

Q5: Prove that the product of two odd numbers is an odd number.

Use $2x+1$ and $2y+1$	Odd $(2x+1)$ and Odd $(2y+1)$
$(2x+1)(2y+1)$	Multiply
$4xy+2x+2y+1$	Simplify
$(4xy+2x+2y)+1$	Group
$2(2xy+x+y)+1$	Even +1, so odd.

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Topic #3: Deductive Reasoning - Physics

Q6: The force of gravity, ***F***, between two objects that are distance ***r*** apart can be modeled by the equation:

$$F = \frac{100}{(r)^2}$$

Prove that when the distance is tripled, the force is 1/9th of the original amount. *(2 marks)*

Math	*Explanation*
$F = \frac{100}{(3r)^2}$	Triple distance.
$F = \frac{100}{9r^2}$	$(ab)^n = a^n b^n$, so $(3r)^2 = 9r^2$
$F = \frac{1}{9}\left(\frac{100}{r^2}\right)$	Factor out 1/9th. So $\frac{1}{9}$th of original amount.

Q7: The Kinetic Energy, ***E***, of a particular object depends upon its mass, ***m***, and its speed, ***v***, per the equation below:

$$E = \frac{1}{2}m(v)^2$$

Prove that when the speed is halved, the kinetic energy is one quarter of the original amount. *(2 marks)*

Math	*Explanation*
$E = \frac{1}{2}m\left(\frac{1}{2}v\right)^2$	Half the speed.
$E = \frac{1}{2}m\left(\frac{1}{4}v^2\right)$	$(ab)^n = a^n b^n$, so $\frac{1}{2}v^2 = \frac{1}{4}v^2$
$E = \frac{1}{4}\left(\frac{1}{2}mv^2\right)$	Bring $\frac{1}{4}$ out in front. So $\frac{1}{4}$ of original amount.

KEY

Topic #4: Deductive Reasoning – Invalid Proofs

Q8: Hossai is trying to prove the following number trick:

Choose any number. Add 3. Double it. Add 4. Divide by 2. Take away the number you started with.

Each time Hossai tries the trick, she ends up with 5. Her proof, however, does not give the same result.

Hossai's Proof (as a "2-Column Proof")

n	Choose any number
n + 3	Add 3
2n + 6	Double it
2n + 10	Add 4
2n + 5	Divide by 2
n + 5	Take away the number you started with

Where is the error in Hossai's proof?

When you divide $2n+10$ by 2, you get $n+5$

Q9: Mr. Bayer believes that he just proved that 2 = 1.

Bayer's Proof (as a "2-Column Proof")

Let a = b	Set a = b
$a^2 = ab$	Multiply both sides by a
$a^2 - b^2 = ab - b^2$	Subtract b^2 from both sides
$(a + b)(a - b) = b(a - b)$	Factor each side
a + b = b	Divide both sides by (a-b)
b + b = b	Replace a with b, since a = b
2b = b	Collect like terms on left side
2 = 1	Divide both sides by b

Where is the error in Bayer's proof?

If $a = b$ then dividing by $(a-b)$ is the same as dividing by zero.

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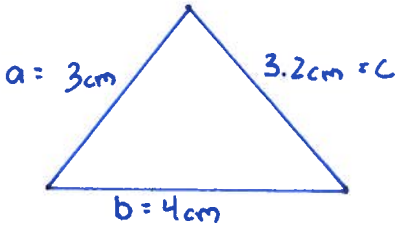
Q10: Mr. Bayer claims that the cube of a number is always ***larger*** than the number itself. Give one counter-example to this claim. *(1 mark)*

$0.5^3 = 0.125$, which is smaller than 0.5

Q11: Mr. Bayer makes the claim that Pythagoras Theorem ($a^2 + b^2 = c^2$) works for every triangle. Using a ruler, create one counter-example where this is not true. Label the lengths of each side. *(1 mark)*



Only works for 90° triangles.

Q12: Mr. Bayer decides to show that the product of an odd number and 3 is always an odd number. However, he made a mistake somewhere. Find which line has the mistake, and correct his proof. *(3 marks)*

Line	Math	Explanation
1	Let (2x+1) be my odd number.	Even number plus one, which makes it odd.
2	(2x+1)(3)	Multiply my numbers
3	6x+4	Simplify
4	2(3x+2)	Hold on... this is divisible by 2. Did I make a mistake?

Line	Math	Explanation
1	Let (2x+1) be my odd number.	Even number plus one, which makes it odd.
2	(2x+1)(3)	Multiply my numbers.
3	6x+3	Simplify.
4	(6x+2)+1	Split and group.
5	2(3x+1)+1	Even number +1 is odd.