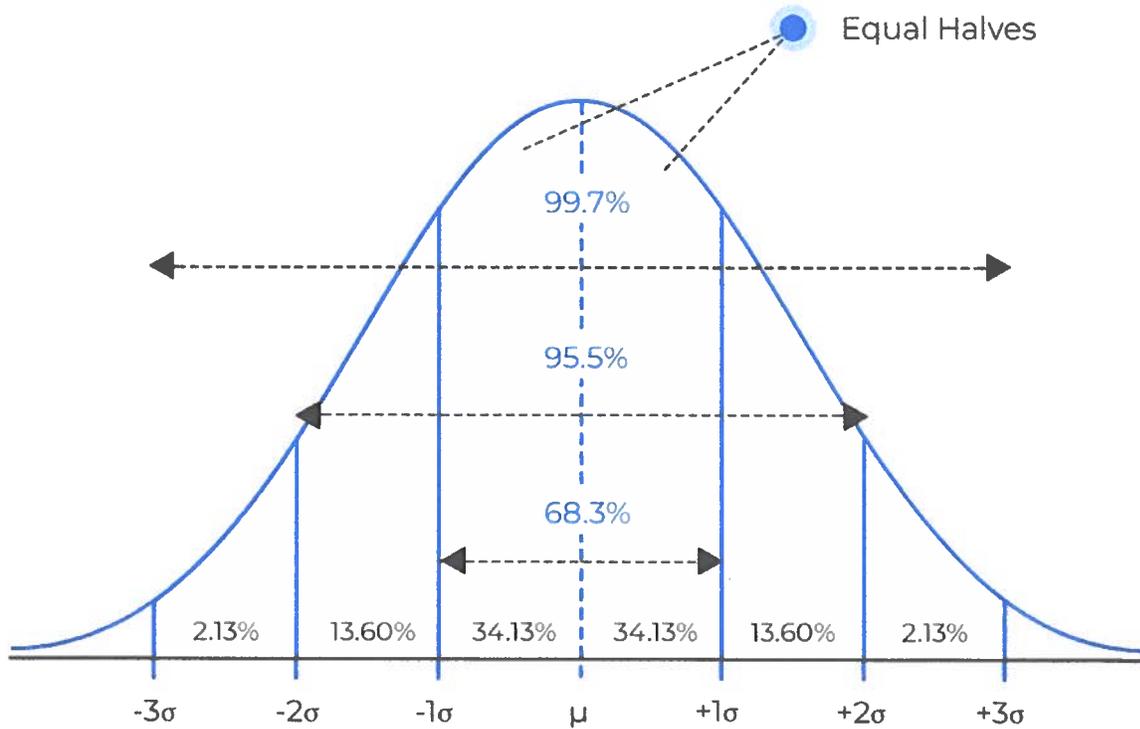


4.1 - z-Scores

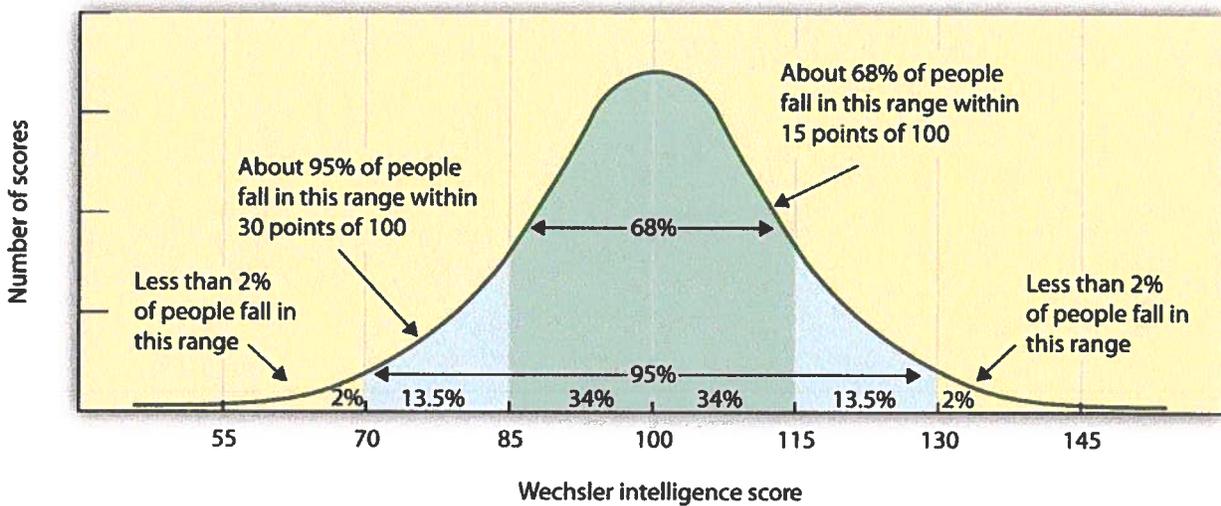
**Part 1 - Review of Normal Distribution**

Below is a *Gaussian Distribution* (Bell Curve, Normal Distribution):



**No. of standard deviations from the mean**

And here is a distribution of a population's intelligence quotients (IQ):



**Part 2 – What is a z-score? How do we use a z-score table?**

A z-score can best be described as “how many” standard deviations a value is away from the mean. Using IQ as an example, let's work with the accepted mean of 100 and standard deviation of 15.

Assuming someone has an IQ of 130, we can see that means they are 2 standard deviations above the mean.

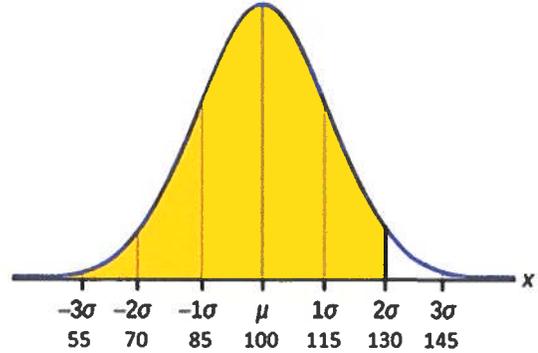
$$x = \mu + z\sigma$$

$$130 = 100 + z(15)$$

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{130 - 100}{15}$$

$$z = 2$$



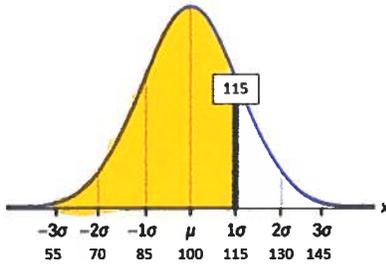
When using a z-score table, keep in mind that this identifies the percentage of the population that is below this value.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
	↓		↓		↓		↓		↓	
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890

So 97.72% of the population has an IQ below 130.

**Part 3 – Finding z-Scores from Raw Values**

**Q1:** Given a mean of 100 and a standard deviation of 15, determine the percentage of the population with IQs below (a) 115, (b) 70, and (c) 122.



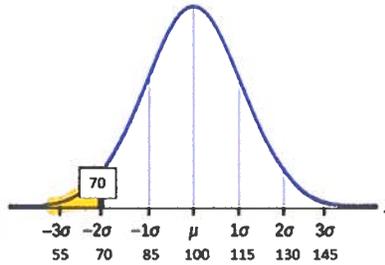
This data point can be represented as  $x = \mu + z\sigma$  or  $115 = 100 + z(15)$

We can solve for z, or "how many" standard deviations the value is away from the mean.

$$z = \frac{x - \mu}{\sigma} \quad \text{or} \quad z = \frac{115 - 100}{15}$$

And we find that  $z = +1$ , so this value is 1 standard deviation above the mean.

Consulting our z-score table, we find that this puts the value above 84.13% of the population.



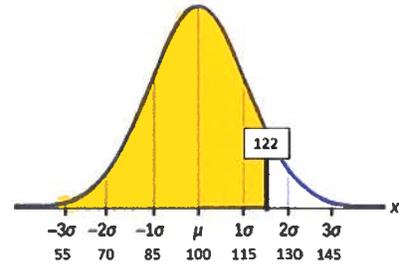
$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{70 - 100}{15}$$

$$z = -2$$

z-score table shows 0.0228

So 2.28% of the population has an IQ below 70.



$$z = \frac{x - \mu}{\sigma}$$

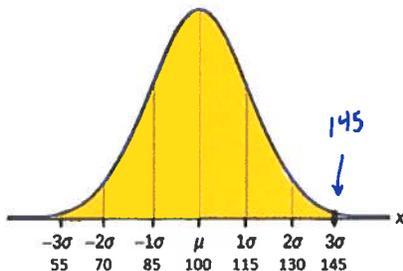
$$z = \frac{122 - 100}{15}$$

$$z = 1.46 \text{ or } 1.47$$

z-score table shows 0.9292

So 92.92% of the population has an IQ below 122.

**Q2:** What percentage of the population has an IQ between 115 and 145?

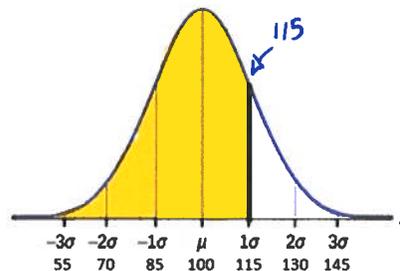


$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{145 - 100}{15}$$

$$z = 3.00$$

So 99.87% of the population below this value.

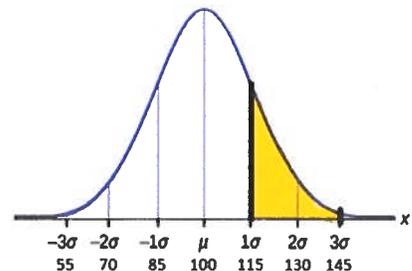


$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{115 - 100}{15}$$

$$z = 1.00$$

So 84.13% of the population below this value.



$$99.87 - 84.13$$

So 15.74% of population fits in this range.